Precoded BICM design for MIMO transmit beamforming and associated low-complexity algebraic receivers

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Abstract— **In this paper, we design bit interleaved coded modulations including a partial algebraic precoder in order to transmit several spatial streams with full diversity over a transmit beamformed MIMO channel. The achievable diversity orders with the coded modulation are derived, and allow for efficiently choosing the system parameters for optimizing the performance. Additionnally, a very efficient low complexity soft output detector based on algebraic reduction is presented. With a low complexity at the receiver, the precoded BICM system allows achieving high performance at very high efficiencies, e.g., 12 bits per second per hertz.**

I. Introduction

The new standards of wireless communications, e.g., 802.11n and 3GPP-LTE, include a variety of techniques for transmitting over mutliple antenna channels: Spatial-division multiplexing, space-time block coding, transmit beamforming, and so on. A trade-off between data rate, range, mobility and receiver complexity can be achieved by a relevant choice of the transmission technique. In this paper, we focus on low mobility users and transmit beamforming techniques. For low and average spectral efficiencies, the MIMO channel capacity can be maximized by transmitting on the best eigenvalue of the channel realization and can be further improved by applying water-filling techniques [6]. In the opposite, for high spectral efficiencies, the capacity is maximized by transmitting on several eigenvalues of the channel and the improvement obtained by water-filling is negligible. Unfortunately, the singular values of the MIMO channel do not exhibit the same diversity orders, and uncoded systems exhibit performance with a diversity order equal to the lowest diversity order of the selected singular values. Full diversity is achieved when transmitting on the best eigenvalue, but the performance is drastically reduced when increasing the modulation cardinality. From these statements, we propose to design full diversity systems by using a combination of spatial division multiplexing and transmit beamforming. The error correcting code allows for recovering a part of the transmit diversity, and, if needed, a partial algebraic precoder is added to guarantee full diversity.

In section II, we present the channel model and notations. In section III, the equivalent channels as seen by the decoder input and including a partial precoder are described, while achievable bounds on the diversity for these channels are derived in section IV. These bounds are used to choose the optimal size of the partial algebraic precoder. In section V, we describe a low complexity soft output detector associated to the precoder, based on an algebraic reduction. In section VI, we show some simulation results for 4×2 and 3×3 MIMO channels and spectral efficiencies up to 12 bits per second per hertz.

II. Channel model and parameters

We consider a $N_t \times N_r$ MIMO channel with N_t transmit antennas and N_r receive antennas. An OFDM-based transmission is assumed, and the transmit and receive antennas are spatially un-correlated. The channel model is defined by the $N_t \times N_r$ matrix with complex gaussian i.i.d. entries of unity variance. We consider flat fading channels, i.e., the channel coherence bandwidth is much larger than the signalling bandwidth. This asumption is for example realistic in the case of sub-band allocation of a multi-user system. The presented results might easily be extended to systems experiencing higher degrees of frequency diversity, but its main goal is to achieve high data rate even in the worst case of flat fading channels.

The channel is assumed to be known at the transmitter. In practice, this can be achieved for slow time variation of the channel and a feedback of the channel estimation between the receiver and the transmitter, or by using the channel reciprocity of TDD-like transmissions, under the asumption of hardware or software calibrated transmission and reception Radio-Frequency paths at the transmitter. Under the channel knowledge asumption, the channel matrix $\mathbf{H}$ is classically decomposed into its singular values representation

$$\mathbf{H} = \mathbf{V}_1 \mathbf{D} \mathbf{V}_2 \tag{1}$$

where $\mathbf{D}$ is a $N_t \times N_r$ diagonal matrix wich diagonal coefficients α_i are the $\min(N_t, N_r)$ non-null singular values of $\mathbf{H}$. The singular vectors matrices $\mathbf{V}_1$ and $\mathbf{V}_2$ are unitary, of size $N_t \times N_t$ and $N_r \times N_r$, respectively. The singular values are sorted in decreasing order of magnitude $i \leq j, \alpha_i \geq \alpha_j$.

The number of spatial streams is by definition equal to the number $N_s \leq \min(N_t, N_r)$ of singular values selected by the transmitter, and the transmit beamforming $N_s \times N_t$ matrix $\mathbf{T}$ is built from the N_s first rows of $\mathbf{V}_1^\dagger$. Let $\mathbf{\Delta}$ define the $N_s \times N_s$ equivalent diagonal channel matrix which diagonal

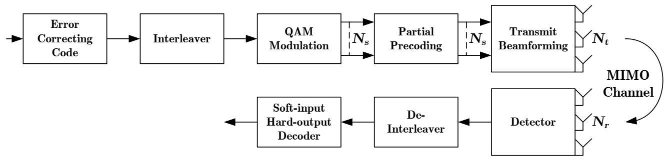

Fig. 1. Transmitter and Receiver structure

coefficients are the N_s selected singular values $\alpha_1, \ldots, \alpha_{N_s}$ of the MIMO channel $\mathbf{H}$.

The transmitter has a bit interleaved coded modulation (BICM) structure including a partial algebraic linear precoder as described in Fig. 1. The information word $\mathbf{b}$ of length L, is converted into a codeword $\mathbf{c} \in \mathcal{C}$ from a binary error correcting code $\mathcal{C}$ of rate R_c. This codeword is interleaved by an interleaver $\mathcal{I}$ and given to a modulator that converts blocks of $m.N_s$ bits into a vector $\mathbf{z} \in \mathcal{Q}^{N_s}$ of N_s modulation symbols taken from the complex constellation $\mathcal{Q}$ of size 2^m. Each vector $\mathbf{z}$ is multiplied by a linear precoder $\mathbf{\Phi}$ defined as follows:

$$\mathbf{\Phi} = \begin{bmatrix} \mathbf{\Phi}' & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_{N_s - s} \end{bmatrix} \times \mathbf{P} \tag{2}$$

where $\mathbf{\Phi}'$ is a $s \times s$ algebraic precoding matrix that precodes the modulation symbols $z_1, \ldots, z_s$ together into so-called precoded symbols. The other $N_s - s$ non-precoded symbols remain unchanged. The matrix $\mathbf{P}$ is a permutation matrix allowing to configure how to transmit the precoded and non-precoded symbols onto the channel singular values. The choice of the partial precoding strategy and the precoding size s will be further described in section IV. The transmitted information rate is equal to $R = R_c N_s m$ bits per channel use.

From equation 1, and assuming that the transmit beamforming filter $\mathbf{T}$ is applied, we can write the equivalent transmission model as follows

$$\mathbf{y} = \mathbf{z}\mathbf{\Phi}\mathbf{\Delta} + \boldsymbol{\eta} \tag{3}$$

where $\mathbf{y}$ is the equivalent received vector and $\boldsymbol{\eta}$ is a N_s-dimensional complex gaussian noise of variance N_0 per real-dimension.

The receiver structure is illustrated in Fig. 1. The detector computes a soft information based on the knowledge of $\mathbf{H}$ and the received vector $\mathbf{y}$. The channel decoder then computes the estimation $\hat{\mathbf{b}}$ based on the de-interleaved soft information coming from the detector and using a soft-input hard-output decoder, such as the Viterbi algorithm [7] for convolutional codes. Indeed, for complexity and lattency reasons, we choose not to allow iterative processing between the detector and the decoder at the receiver. We believe that the gain brought by iterative joint detection and decoding is low for this system and the performance comparisons are left for further studies.

Thanks to the diagonal shape of the equivalent channel matrix $\mathbf{\Delta}$, no interference is observed on non-precoded symbols. Consequently, the non-precoded symbols are detected independently by low complexity mono-dimensional soft output detectors.

However, in order to exploit the diversity brought by algebraic linear precoding, the precoded symbols must be detected alltogether. First, we will consider that an exhaustive marginalization is used for the joint detection of precoded symbols. Then, we will propose a low complexity soft output-receiver that guarantees the recovering of the diversity order brought by the linear precoder.

III. Equivalent binary-input Binary-output channel model

The concatenation of the error-correcting code, the interleaver, the modulation, the partial precoding, the transmit beamforming scheme and the MIMO channel defines a euclidean code $\mathcal{R}$ that associates 2^L information words $\mathbf{b}$ to 2^L points $\mathbf{Z\Phi TH} \in \mathcal{R} \subset \mathbb{C}^{LN_r/R}$, where $\mathbf{Z}$ is the concatenation of the vectors $\mathbf{z}$ of size N_s transmitted during the L/R time periods.

The global maximum likelihood receiver finds the information word that minimizes the euclidean distance between the transmitted codeword of $\mathcal{R}$ and the received vector. The maximum likelihood performance is classically upper bounded by the union bound, i.e., by the sum of all possible pairwise error probabilities between two codewords of the euclidean code. If all pairwise error probabilities exhibit the full diversity order after averaging over several channel realization, then the maximum likelihood performance also exhibits the full diversity order. Let us assume that the binary codeword $\mathbf{c} \in \mathcal{C}$ is transmitted and a different binary codeword $\mathbf{c}'$ is decoded. The Hamming weight w of the error event $\mathbf{c} - \mathbf{c}'$ is greater than d_{Hmin}, the minimal Hamming distance of the error correcting code $\mathcal{C}$. The probability of such an error event is minimized with the euclidean square distance $\|\mathcal{F}(\mathbf{c}) - \mathcal{F}(\mathbf{c}')\|^2$ where $\mathcal{F}(.)$ is the function that converts codewords of $\mathcal{C}$ into codewords of $\mathcal{R}$.

Let us now asume that for any pair of codewords, all the non-null bits of $\mathbf{c} - \mathbf{c}'$ are transmitted in different time periods, which can be achieved thanks to a good interleaver. Thus, $\|\mathcal{F}(\mathbf{c}) - \mathcal{F}(\mathbf{c}')\|^2 = \sum_{k=1}^{w} d_k^2$ where $d_k^2 = \|(\mathbf{z}_k - \mathbf{z}'_k)\mathbf{\Phi}\mathbf{\Delta}\|^2$ and $\mathbf{z}_k$ and $\mathbf{z}'_k$ are the transmitted and decoded vectors of symbols in the time periods associated to the non null-bits of $\mathbf{c} - \mathbf{c}'$. If a Gray mapping is used in the modulation $\mathcal{Q}$, the vector $\mathbf{z}_k - \mathbf{z}'_k$ has only one non-null entry out of N_s. By consequence, when computing the pairwise error probabilities, the coded bits appear to be modulated by a scaled BPSK modulation and sent through one out of N_s blocks. The distances of the equivalent BPSK are dependent of the modulation size. Each of the N_s blocks corresponds to the transmission over a $1 \times N_s$ channel $\mathbf{\Phi}_\ell \mathbf{\Delta}$, where $\mathbf{\Phi}_\ell$ is the ℓ-th row of $\mathbf{\Phi}$. The N_s-block channel defines the binary-input binary-output equivalent channel as seen between the binary encoder output and decoder input. In the next section, we will derive the bound on the diversity achieved by error correction coding over the binary-input binary-output equivalent channel associated to partially precoded transmit beamforming MIMO channels.

IV. Bounds on the diversity of the partially precoded scheme

In order to establish the bounds of the coded modulation over the partially precoded transmit-beamformed MIMO channel, we will first recall some results from previous work in [2] and needed for this study.

A. Matryoshka channels and associated bounds

Let us consider N independent fading random variables. Let $\mathcal{M}(\mathcal{D},\mathcal{L})$ be a channel built from the concatenation of $N_s = |\mathcal{D}|$ blocks, where $\mathcal{D}$ and $\mathcal{L}$ are the sets of diversity order and lengths of each block, respectively. The integer $|\mathcal{D}|$ is the cardinality of $\mathcal{D}$. The i-th diversity block is defined by a combination of a subset $\mathcal{S}(i)$ of $\mathcal{D}(i) \leq N$ random variables, such that $\mathcal{S}(i+1) \subset \mathcal{S}(i)$, i.e. the blocks are sorted such that $\forall i < j, \mathcal{D}(i) > \mathcal{D}(j)$ and we assume that $\mathcal{D}(1) = N$ has the highest diversity order. The maximal diversity observed after decoding a rate-R_c code transmitted over a $\mathcal{M}(\mathcal{D},\mathcal{L})$ channel is $\mathcal{D}(i)$ where i is given by the following inequalities:

$$\sum_{k=1}^{i-1} \mathcal{L}(k) < R_c \sum_{k=1}^{|\mathcal{D}|} \mathcal{L}(k) \leq \sum_{k=1}^{i} \mathcal{L}(k) \tag{4}$$

and is achievable for any linear code[1]. Furthermore, if the code is systematic, the bound is achieved by sending the information bits on the blocks of highest diversity, which puts conditions on the design of the interleaver.

B. Bound on the diversity for the non-precoded case

If no precoding is used then $\mathbf{\Phi} = \mathbf{I}$ and the N_s blocks are defined by the N_s singular values $\alpha_1, \cdots, \alpha_{N_s}$. Since we use an ordered singular value decomposition, it can be shown that the diversity associated to the i-th singular value is equal to $(N_t-i+1)(N_r-i+1)$. Furthermore, if $\mathcal{S}$ is defined as the set of the N_tN_r independent random variables $h_{i,j}$ observed in the MIMO channel $\mathbf{H}$, then the singular value α_i is defined from a subset $\mathcal{S}_i$ of $(N_t-i+1)(N_r-i+1)$ independent variables. Furthermore, it can be shown that $\forall i < j, \mathcal{S}_i \subset \mathcal{S}_j$. Thus, the equivalent block fading channel associated to the transmit beamforming techniques with equal modulation on each data stream is a Matryoshka channel $\mathcal{M}(\mathcal{D},\mathcal{L})$ with $0 < i \leq N_s$, $\mathcal{D}(i) = (N_t-i+1)(N_r-i+1)$ and $\mathcal{L}(i) = L/(R_cN_s)$. The maximum diversity $\delta_1^{(1)}$ achievable with a code of rate $R_c > 0$ is equal to

$$\delta_1^{(1)} = (1 + N_t - \lceil R_cN_s \rceil)(1 + N_r - \lceil R_cN_s \rceil) \tag{5}$$

If different modulation cardinalities are used for each spatial stream, the bound on the diversity change and can be easily derived from (4) . This case is out of the scope of this paper and will be addressed in future studies.

[1]Please refer to [2] for a complete proof of this result

C. Bound on the diversity for precoding strategy 1

Let us now assume that the permutation matrix $\mathbf{P}$ is chosen according to the following precoding strategy: the first s symbols out of N_s are precoded together and transmitted over the singular values α_1 and $\alpha_{N_s-s+2}, \cdots, \alpha_{N_s}$; the other N_s-s symbols remain unchanged. The singular value α_1 having the maximum diversity order N_tN_r is precoded along with other singular values that do not bring additional diversity orders.

If the detector associated to the precoded symbols is optimal, the soft information associated to the s precoded symbols carry a diversity order equal to N_tN_r. The soft informations on the other coded bits carry a diversity order equal to the diversity order of the singular value they are transmitted through. Thus, the equivalent block fading channel associated to the transmit beamforming techniques with equal modulation on each data stream is a Matryoshka channel $\mathcal{M}(\mathcal{D},\mathcal{L})$ with $0 < i \leq N_s - s + 1$, $\mathcal{D}(i) = (N_t - i + 1)(N_r - i + 1)$, $\mathcal{L}(1) = sL/(R_cN_s)$ and $\mathcal{L}(i > 1) = L/(R_cN_s)$. From (4), the maximum diversity $\delta_s^{(1)}$ achievable with a code of rate $R_c > 0$ is equal to

$$\delta_s^{(1)} = \min\left(N_tN_r, (s + N_t - \lceil R_cN_s \rceil)(s + N_r - \lceil R_cN_s \rceil)\right) \tag{6}$$

D. Bound on the diversity for the precoding strategy 2

Let us now assume that the permutation matrix $\mathbf{P}$ is chosen according to the following precoding strategy: the first s symbols out of N_s are precoded together and transmitted over the singular values $\alpha_1, \cdots, \alpha_s$; The equivalent block fading channel associated to the transmit beamforming techniques with equal modulation on each data stream is a Matryoshka channel $\mathcal{M}(\mathcal{D},\mathcal{L})$ with $0 < i \leq N_s - s + 1$, $\mathcal{D}(1) = N_tN_r$, $\mathcal{D}(i > 1) = (N_t-s-i+2)(N_r-s-i+2)$, $\mathcal{L}(1) = sL/(R_cN_s)$ and $\mathcal{L}(i > 1) = L/(R_cN_s)$. The maximum diversity $\delta_s^{(2)}$ achievable with a code of rate $R_c > 0$ is equal to

$$\delta_s^{(2)} = \begin{cases} N_tN_r, & 0 < R_c \leq s/N_s \\ \delta_1^{(1)}, & R_c > s/N_s \end{cases} \tag{7}$$

E. Transmitter design from the bounds

In our transmission scheme, the precoder introduces interference between s modulation symbols in order to exploit a part of the transmit diversity in the detector computation. However, as we do not consider iterative joint detection and decoding, this amount of interference is not cancelled and degrades the coding gain. Furthermore, if we consider a detection of the precoded symbols based on an exhaustive marginalization over the list of $\mathcal{Q}^s$ possible points, the complexity grows exponentially with s. It is then preferable to choose the minimal parameter s that allows to recover the full diversity order.

From the bounds on the diversity orders (6) and (7), we conclude that: If $R_c \leq 1/N_s$, no partial precoder is needed to reach the full diversity order. In that case all the diversity is collected by the decoder, and we choose $s = 1$.

If $s = N_s$, the full diversity order is collected at the detector, and the coding rate can be up to $R_c = 1$ without degrading the slope of the performance. As a remark, the bad coding gain of the precoded system would be enhanced by iterative joint detection and decoding.

We observe that $\forall s, \delta_s^{(1)} \geq \delta_s^{(2)}$, and the equality is achieved with $s \geq R_c N_s$. For high coding rates, it is preferable to provide more robustness to systematic bits. Thus, if $s \geq R_c N_s$ and the coding rate is high (e.g., $R_c = 2/3$), we choose precoding strategy 1, else we choose precoding strategy 2.

We have derived a choice of s and $\mathbf{P}$ depending on N_s and R_c that ensures performance achieving the target diversity order. Unfortunately, the complexity of the detectors allowing to exploit the diversity brought by linear precoding is usually exponential in s, which goes against the main advantage of transmit beamforming techniques : the low complexity of the receiver, particularly useful in the downlink of cellular networks. In the next section, we present a new soft output detector based on algebraic reductions allowing to exploit the diversity brought by linear precoding and which complexity is linear with s.

V. A FULL-DIVERSITY LOW COMPLEXITY DETECTOR

If linear precoding is used, linear receivers do not recover the expected diversity order and detectors including at least a lattice basis reduction are needed [5], [4]. Lattice basis reductions such has the LLL algorithm [3] must be processed for each new channel realization. The more static the channel in time and flat in frequency, the lower the overhead in complexity at the receiver, when compared to a linear detector. Alternatively, it has been shown in [1] that algebraic reductions associated to a particular design of the algebraic precoder can recover the full diversity order and quasi-ML performance over rayleigh fast fading channel, i.e. over diagonal fading channel. We derive a soft output from a small list of point, built in the reduced lattice (see e.g [8] for soft-output detector based on lattice reduction). Furthermore, we propose a simple technique to solve the classical problems of constellation boundaries and mapping when using an algebraic reduction.

We denote $\mathbf{z}'$ the vector of modulation symbols to be precoded by the algebraic precoder $\mathbf{\Phi}'$ and transmitted through the diagonal $s \times s$ matrix $\mathbf{\Delta}'$ extracted from $\mathbf{\Delta}$, and $\mathbf{y}'$ the associated received vector. We focus on the input-output relationship

$$\mathbf{y}' = \mathbf{z}'\mathbf{\Phi}'\mathbf{\Delta}' + \boldsymbol{\eta}' \tag{8}$$

where $\boldsymbol{\eta}'$ is an additive white gaussian noise of variance N_0 per real dimension. Following [1], the algebraic construction of the unitary matrix $\mathbf{\Phi}'$ allows to write the relationship

$$\mathbf{\Phi}'\mathbf{\Delta}' = \mathbf{\Phi}'\mathbf{U}\boldsymbol{\xi}\mathbf{\Psi} = \mathbf{T}_u\mathbf{\Phi}'\boldsymbol{\xi}\mathbf{\Psi} \tag{9}$$

where $\mathbf{U}$ is a $s \times s$ diagonal matrix of the embedding of the cyclotomic field unit, $\mathbf{T}_u$ is a $s \times s$ basis change matrix of gaussian integers, and $\boldsymbol{\xi}$ and $\mathbf{\Psi}$ are the diagonal matrix of the amplitude and phase correction. Hence, the channel matrix $\mathbf{\Delta}' = \mathbf{U}\boldsymbol{\xi}\mathbf{\Psi}$ is approximated by a matrix $\mathbf{U}$ which algebraic properties allow the matrix commutation $\mathbf{\Phi}'\mathbf{U} = \mathbf{T}_u\mathbf{\Phi}'$. Let us observe that

$$\mathbf{y}'' = \mathbf{y}'\mathbf{\Psi}^\dagger\boldsymbol{\xi}^{-1}\mathbf{\Phi}'^\dagger = \mathbf{z}'' + \boldsymbol{\eta}'' \tag{10}$$

where $\mathbf{z}'' = \mathbf{z}'\mathbf{T}_u$ belongs to a parallelotopic constellation of the Gaussian integers lattice $\mathbb{Z}[j]^s$. Here, $\boldsymbol{\eta}'' = \boldsymbol{\eta}'\mathbf{\Psi}^\dagger\boldsymbol{\xi}^{-1}\Phi^\dagger$ is an additive gaussian noise vector, entries of which are correlated. A good choice of the matrix $\mathbf{U}$, reduces the noise correlation and allows to achieve quasi-ML performance with a simple per-dimension decision. There exists an isomorphism between the set of matrices $\mathbf{U}$ and its associated logarithmic lattice. The set of matrices $\mathbf{\Delta}'$ is then quantized by the matrices $\mathbf{U}$, and the quantization is done in the logarithmic lattice (see [1] for further details). In practice, the candidate number of matrices $\mathbf{U}$ are very limited and can be computed offline. Once $\mathbf{U}$ is chosen, the reduction matrix $\mathbf{T}_u$ can be found directly with a much smaller complexity than LLL or KZ algorithms. Furthermore, since the channel is known at the transmitter, the choice of $\mathbf{U}$ can be done at the transmitter and signalled at the receiver.

The channel model (10) is orthogonal, we can then classically build a small list of points in the neighbourhood of the received point $\mathbf{y}''$ to accurately compute a soft output on the coded bits. The list is built as follows:

1) Make an estimation $\mathbf{z}'$ of the ML point thanks to the algebraic reduction.
2) List the 2^{2s} points of the fundamental paralellotope the received point $\mathbf{y}''$ belongs to. It is simply achieved from the knowledge of $\mathbf{z}''$ and $sign(\mathbf{z}'' - \mathbf{y}'')$

Once the list is built, one can check if the points $\mathbf{z}''\mathbf{T}_u^{-1}$ belong to the modulation $\mathcal{Q}^s$ and find the binary mapping of these points. If s becomes large, the vector by matrix multiplication $\mathbf{z}''\mathbf{T}_u^{-1}$ might have a large impact on the receiver complexity. Let us define $\mathbf{x}$ the $2s$-length vector of integers obtained after translation of the 2^m-QAM modulation, each entry satisfies $0 \leq x_i \leq 2^{m/2} - 1$. Let $a(\mathbf{x})$ be an integer defined as follows

$$a(\mathbf{x}) = \sum_{i=1}^{2s} x_i K^{i-1} \tag{11}$$

where $K = 2^{m/2+\ell}$. The integer $a(\mathbf{x})$ allows a one-to-one mapping between an extended constellation of K^{2s} points and the set of integers $[0..K^{2s} - 1]$. We can compute the integer $a(\mathbf{x})$ from a point $\mathbf{x}'$ in the reduced basis as

$$a(\mathbf{x}) = \sum_{i=1}^{2s}\sum_{j=1}^{2s} x'_j \mathbf{T}_u^{-1}(j,i)K^{i-1} = \sum_{j=1}^{2s} x'_j \mathbf{A}_j \tag{12}$$

where $\mathbf{A}_j = \sum_{i=1}^{2s} \mathbf{T}_u^{-1}(j,i)K^{i-1}$ can be computed offline and stored, as well as $\mathbf{U}$. Finally, the points $\mathbf{x} \in \mathcal{Q}^s$ can be found by

$$x_i = \min(\mathbf{x}'\mathbf{A}/K^i, M) \tag{13}$$

which can be easily implemented by selection of $s.m$ bits in the binary representation of $\mathbf{x}'\mathbf{A}$. For high spectral efficiencies, the received point is close to or inside the constellation boundaries with a good probability. Then if ℓ is high enough

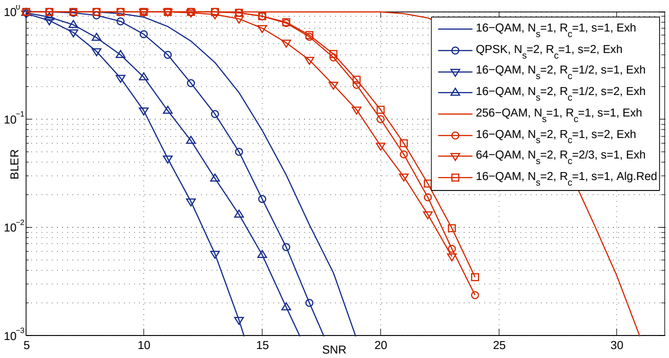

Fig. 2. 4×2 MIMO channel, $L = 864$ information bits, punctured $(133, 171)_8$ convolutional code, 4 and 8 bits per second per hertz.

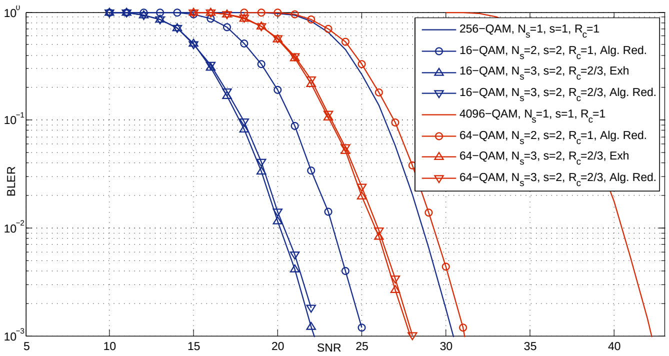

Fig. 3. 3×3 MIMO channel, $L = 864$ information bits, punctured $(133, 171)_8$ convolutional code, 8 and 12 bits per second per hertz.

(typically 2 to 4), the probability of mis-decoding by using this technique is low. If all the points in the list have the same bit value in n positions of the binary labelling, we add n points, each one being created by complementing one of the considered n bits in the labelling of the closest point in the reduced lattice. This step allows for adding stability in the numerical computation of the soft values. In the worst case, the list contains less than $4^s + m$ points which is much lower for high spectral efficiencies than the length 2^{ms} of the exhaustive list of points. Finally, the likelihood of each point is calculated and a marginalization is processed for computing the soft outputs.

VI. Simulation results and conclusions

In the following simulations, we only focus on full-diversity systems. However, this paper can also allow to design systems for a target diversity order inferior to the full diversity order. The block error rate (BLER) is plotted as a function of the SNR=$1/N_0$, i.e., the total transmit power is assumed equal to 1. The possible detectors for the precoded symbols are an exhaustive marginalization 'Exh' or a soft-output detection based on algebraic reduction 'Alg. Red.'. We consider an information word of $L = 864$ information bits, encoded by a punctured $(133, 171)_8$ convolutional code. The interleaver is designed to guarantee that the expected diversity orders are achieved. We use linear precoders of size $s = 2$, defined by

$$\mathbf{\Phi}' = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ (1+j)/\sqrt{2} & -(1+j)/\sqrt{2} \end{bmatrix} \qquad (14)$$

and the associated possibles pre-computed quantized channels $\mathbf{U}$ have diagonal values equal to $[1; 1]$, $[-2.4142; 0.4142]$ and $[-0.4142; 2.4142]$. In Fig. 2, we consider a 4×2 MIMO channel, with transmission schemes achieving the full-diversity order 8, as expected from the theory, and spectral efficiencies of 4 (blue) and 8 (red) bits per second per hertz. We first compare an uncoded 16-QAM ($R_c = 1$) transmitted over the best singular value of the channel ($N_s = 1$) to an uncoded two-streams QPSK ($N_s = 2$), linearly precoded ($s = 2$), and coded $R_c = 1/2$ two-stream 16-QAM modulation with and without linear precoding. We observe that precoded multi-streams transmissions outperform the mono stream transmission. Additionnally, the error correcting code structure brings additional coding gain. We also observe that, without iterative joint detection and decoding, the linear precoding degrades the coding gain. We also have simulated 8-bits per second per hertz spectral efficieny transmissions. There are 6-dB gains between the mono-stream 256-QAM and the linearly precoded 2-stream 16-QAM, which performs as good as the 3-streams 64-QAM with $R_c = 2/3$. We also observe that the hard output algebraic reduction perform very close to the optimal detector. In Fig. 3, we consider a 3×3 MIMO channel, with transmission schemes achieving the full-diversity order 9 and spectral efficiencies of 8 (blue) and 12 (red) bits per second per hertz. More than 14 dBs are gained when comparing a mono-stream, yet unpractical, 4096-QAM and a coded $R_c = 2/3$ 3-streams 64-QAM with precoding $s = 2$. We can also observe how close the algebraic reduction performs when compared to the optimal MAP detector. In this case, the number of points in the list is in average 24 instead of 2^{18}.

Finally, the presented simulation results confirm the analysis developped in the paper: Very high spectral efficiencies, full diversity and high coding gain can be achieved with coded and precoded transmit beamforming. While keeping a very low complexity receiver by the mean of an algebraic reduction, one can achieve spectral efficiencies higher than 10 bits/s/Hz with high robustness on flat channels. Future studies will mesure the impact of this technique from a system level point of view.

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