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Rate-splitting for polar codes on block fading channels without CSIT

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Rate-Splitting for Polar Codes on Block Fading Channels without CSIT

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***Abstract*—This paper presents a polar code design for block fading channels when no channel state information is available at the transmitter, which involves that the frozen bits cannot be changed dynamically with the fading realizations. An outer parallel code is concatenated with an inner polarization kernel that changes the properties of the block fading channel. The rate-splitting between the parallel outer codes is optimized for minimizing the system's outage probability. We show that when polar codes are used as component outer parallel codes, the overall coding structure is a polar code whose frozen bits are designed according to the rate-splitting optimization. It is shown that this scheme achieves quasi-optimal performance at high throughput with ARQ mechanisms.**

I. Introduction

The block fading channel is a simple and relevant model for designing error correcting codes for wireless channels with no Channel State Information at the Transmitter (CSIT). It allows characterizing the diversity order that can be exploited by the error correcting code according to its coding rate [1]. When considering the widely used bit interleaved coded modulations [2], the equivalent channel seen between the output of the error correcting code and the input of its decoder falls back to a block fading channel with binary input in a large variety of cases including QAM modulations, multiple antenna [3], or orthogonal frequency division multiplexing. The design of error correcting codes on block fading channels have already been widely investigated and mainly relies on the multiplexing of the coded bits on the independent fading blocks [4]. However, the proposed schemes are neither flexible in terms of rate or code size, nor proven to achieve close to the outage probability performance.

The polar codes [5] have been introduced as the first coding schemes asymptotically reaching the capacity of a Binary Discrete Memoryless Channel (BDMC). Several studies consider the optimization of polar codes for (block-) fading channels [6], high rate modulations and MIMO channels (see, e.g., in [7]), but require the knowledge of CSIT. Indeed, for a BDMC with fixed parameter (e.g., the Signal to Noise Ratio (SNR) of an Additive White Gaussian Noise (AWGN) channel), a polar code is designed by the choice of its frozen bits. This choice is usually based on a sorting of the mutual information between the input of the polar codes and the channel output, which changes with the BDMC parameter and requires CSIT.

In [8], the authors propose a hierarchical coding scheme designed for fading binary symmetric channels that does not require CSIT. However, this coding structure is unsuitable when the number of fading blocks of the channel is small or when a large number of quantization levels is required in order to accurately represent the fading random variables. In this paper, we specifically address the Rayleigh block fading channels with a low number of blocks by performing a statistical optimization of the transmission scheme and frozen bits selection.

Firstly, in Section II, the block fading channel and the utility functions for long-term link adaptation are presented. Secondly, Section III introduces the parallel coding strategy for the polarized block fading channel. Then, Section IV focuses on the $N = 2$ block fading channel case with a size-2 polarization kernel. The optimization problem for the rate-splitting is described and an approximation that eases its computation is derived. Finally, the application of polar codes as constituent parallel codes is discussed in Section V, simulation results are shown in Section VI and a conclusion is given in Section VII.

II. System model

Let $I(X_i; Y_i)$ be the mutual information between the binary input X_i and output Y_i of a BDMC. It can be expressed as the mathematical expectation of a function of the Log-Likelihood Ratio (LLR) L_i of the correct versus incorrect decisions on the binary symbol X_i, such that

$$I(X_i; Y_i) = 1 - E\left[\log_2(1 + e^{-L_i})\right]. \tag{1}$$

Let us consider the transmission of a coded information message over a block fading channel with N transmission blocks comprising the same number of transmission resource elements. The i-th block is experiencing a quasi-static random fading channel coefficient α_i, which is constant within the block and changes independently from one information message transmission to another. The transmission of each part of the coded information message is performed via a BPSK modulation and suffers from an AWGN at the receiver, defining a long-term SNR $\gamma = E_s/2N_0$. Thus, the instantaneous SNR $\rho_i = \gamma|\alpha_i|^2$ for the i-th block is a random variable. It can be estimated at the receiver, which involves that the LLR of the correct versus incorrect decisions from the observation Y_i on an AWGN channel with BPSK input and SNR ρ_i is $L_i = 4\rho_i Y_i \sim \mathcal{N}(4\rho_i, 8\rho_i)$. The mutual information associated to the i-th transmission block is a non decreasing function $\mathcal{I}(.)$ of ρ_i, denoted $\mathcal{I}(\rho_i) = I(X_i; Y_i)$.

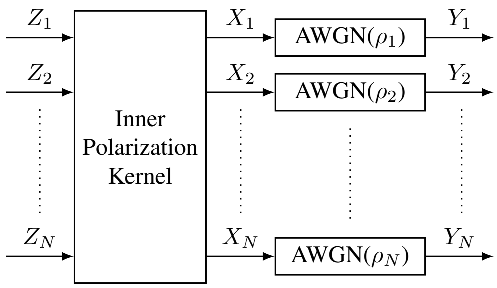

Fig. 1. Block fading channel with an inner polarization kernel.

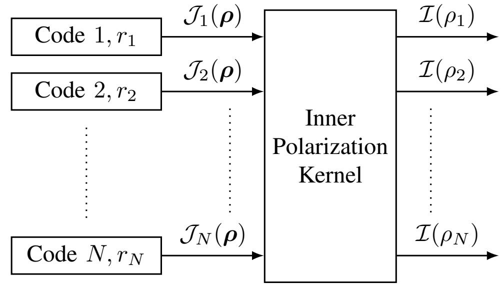

Fig. 2. Outer parallel coding with rate-splitting and inner polarization kernel.

We consider that the instantaneous SNRs ρ_i are unknown at the transmitter, and the relevant information theoretic metric for the channel is the outage probability defined by

$$P_{out}(R,\gamma) = \mathrm{P}\left(\sum_i \mathcal{I}(\rho_i) < NR\right), \qquad (2)$$

where R is the average transmission rate. We assume that the transmitter only knows the joint probability density function $p_\gamma(\rho_1,\ldots,\rho_N)$, parametrized by the long term SNR γ. The transmission strategy can be determined by optimizing the outage probability $P_{out}(R,\gamma)$ for broadcast services or the throughput $R(1-P_{out}(R,\gamma))$ for Automatic Repeat reQuest (ARQ) retransmission schemes.

Fig. 1 shows the channel model including a Rate-1 inner polarization kernel that combines the independent inputs $[Z_1,\ldots,Z_N]$ into $[X_1,\ldots,X_N]$. The conditional mutual information $\mathcal{J}_i(\boldsymbol{\rho}) = I(Z_i;\mathbf{Y}|\mathbf{Z}_1^{i-1})$, where $\mathbf{Y} = [Y_1,\ldots,Y_N]$ and $\mathbf{Z}_1^{i-1} = [Z_1,\ldots,Z_{i-1}]$, is a function $\mathcal{J}_i(.)$ of the instantaneous SNRs set $\boldsymbol{\rho} = [\rho_1,\ldots,\rho_N]$. The purpose of our study is to evaluate the impact of the inner polarization kernel on the statistical robustness to random fluctuations of the instantaneous SNR. Thus, we propose the definition:

Definition 1. *A quasi-static BDMC is defined by a BDMC with a mutual information $\mathcal{J}(\boldsymbol{\rho})$, where $\boldsymbol{\rho}$ is a set of parameters randomly changing from one information message transmission to another.*

For example, the SNR is the parameter of the AWGN BDMC, and applying a quasi-static fading defines its quasi-static counterpart.

The channel model defined by the use of an inner polarization kernel over a block fading channel results in a set $\{\mathcal{J}_1(\boldsymbol{\rho}),...,\mathcal{J}_N(\boldsymbol{\rho})\}$ of equivalent parallel quasi-static BDMCs. These parallel BDMCs do not necessarily have identically distributed and independent parameters and the equivalent BDMCs are not necessarily AWGN channels even when the ones of the initial block fading channel are. The inner polarization kernel correlates and changes the fading distribution and the nature of the BDMCs. The properties of the new parallel channel model and the optimization of the transmission scheme for this model will be discussed in the next sections.

III. Outer parallel coding strategy

The capacity conservation of parallel BDMC channels encoded by a polarization Kernel is one of the main result of channel polarization [5]. It is achievable under a proper design of the Kernel and under successive decoding. This involves that $\sum_i \mathcal{J}_i(\boldsymbol{\rho}) = \sum_i \mathcal{I}(\rho_i)$ and consequently that the outage probability remains unchanged under the effect of a polarization kernel:

$$P_{out}(R,\gamma) = \mathrm{P}\left(\sum_i \mathcal{J}_i(\boldsymbol{\rho}) < NR\right). \qquad (3)$$

The outage probability is upper bounded by $P_{out}(R,\gamma) \leq P_{ub}(\mathbf{r},\gamma)$ for all $\mathbf{r} = [r_1,\ldots,r_N]$ such that $\sum_i r_i = NR$, and

$$P_{ub}(\mathbf{r},\gamma) = \mathrm{P}\left(\bigcup_i \left(\mathcal{J}_i(\boldsymbol{\rho}) < r_i\right)\right). \qquad (4)$$

This upper bound independently considers the outage events $\mathcal{J}_i(\boldsymbol{\rho}) < r_i$ of the equivalent N parallel BDMCs resulting from the inner polarization kernel. Also, the average rate R is split into sub-rates r_i/N. Thus, the upper bound is associated with a coding strategy comprising an outer parallel coding stage concatenated with an inner polarization kernel, as illustrated in Fig. 2, where the associated (conditional) mutual information values are shown on the parallel links. This coding strategy is equivalent to the diagonal multiplexing of [6]. The performance degradation induced by the outer parallel coding approach are linked to the events when $(\sum_i \mathcal{J}_i(\boldsymbol{\rho}) \geq NR)\bigcap(\bigcup_i (\mathcal{J}_i(\boldsymbol{\rho}) < r_i))$. As a remark, a theoretical way to reach optimality without CSIT is the broadcast approach with superposition coding involving an infinite (or sufficiently large) number of levels [9]. This approach is also optimal without any polarization of the block fading channel, but does not meet the practical requirements towards a good performance/complexity trade-off. We will first prove the achievability of $P_{ub}(\mathbf{r},\gamma)$ in Proposition 1, and the interest of using an inner polarization kernel in Proposition 2.

Definition 2. *The set $\mathcal{C}(r,\mathcal{J}(\boldsymbol{\rho}))$ contains the error correcting codes with rate r achieving the outage probability of a quasi-static BDMC with associated mutual information $\mathcal{J}(\boldsymbol{\rho})$.*

In other words, when a code belongs to $\mathcal{C}(r,\mathcal{J}(\boldsymbol{\rho}))$, it is error-free when $\mathcal{J}(\boldsymbol{\rho}) \geq r$.

Proposition 1. *In an outer parallel coding and inner polarization kernel concatenation, if each i-th outer parallel code is outage achieving, i.e., belongs to $\mathcal{C}(r_i, \mathcal{J}_i(\boldsymbol{\rho}))$, the outage probability $P_{ub}(\mathbf{r}, \gamma)$ is achievable.*

Proof. Let $\boldsymbol{\rho}$ be such that $\forall i, \mathcal{J}_i(\boldsymbol{\rho}) \geq r_i$. Each conditional mutual information $\mathcal{J}_i(\boldsymbol{\rho}) = I(Z_i; \mathbf{Y}|\mathbf{Z}_1^{i-1})$ is obtained by the polarization kernel effect under the condition of correct decoding of $\mathbf{Z}_1^{i-1}$, i.e., of the parallel codes 1 to $i-1$. By induction, if the codes 1 to $i-1$ have been correctly decoded, the i-th code observes a BDMC of capacity $\mathcal{J}_i(\boldsymbol{\rho}) \geq r_i$ which involves a correct decoding (otherwise, the code does not belong to $\mathcal{C}(r_i, \mathcal{J}_i(\boldsymbol{\rho}))$). Thus, the intersection of the events $\bigcap_i(\mathcal{J}_i(\boldsymbol{\rho}) \geq r_i)$ leads to no error, taking the complement gives the proof. □

Proposition 2. *An inner polarization kernel with associated set $\{\mathcal{J}_i(\boldsymbol{\rho})\}$ of non-decreasing component-wise conditional mutual information functions decreases the outage probability of the parallel coding scheme on a block fading channel.*

Proof. The proof is given by showing that the non-outage events of the parallel coding scheme sent directly on the block fading channel with any rate-split $(r'_1, \dots, r'_N)$ are also non-outage events when the inner polarization Kernel is used with a rate-split $\mathbf{r} = (\mathcal{J}_1(\boldsymbol{\rho}'), \dots, \mathcal{J}_N(\boldsymbol{\rho}'))$ such that $\boldsymbol{\rho}' = (\mathcal{I}^{-1}(r'_1), \dots, \mathcal{I}^{-1}(r'_N))$. First, the capacity conservation property of the polarization kernel involves that both schemes have the same rate $\sum_i \mathcal{J}_i(\boldsymbol{\rho}') = \sum_i r'_i$. Let $\boldsymbol{\rho}$ be a random fading realization leading to a non-outage event for the parallel coding scheme on the block fading channel with no inner polarization kernel, i.e., $\forall j, \mathcal{I}(\rho_j) \geq r'_j$. For all i, the conditional mutual information $\mathcal{J}_i(\boldsymbol{\rho})$ being non-decreasing component-wise, and the function $\mathcal{I}$ being non-decreasing, we get $\forall j, \rho_j \geq \mathcal{I}^{-1}(r'_j) \Rightarrow \mathcal{J}_i(\boldsymbol{\rho}) \geq \mathcal{J}_i(\boldsymbol{\rho}')$, which leads to a non-outage event with the inner polarization Kernel. □

Thus, by assuming that the component codes of the outer parallel code are outage-achieving, the inner polarization kernel can only improve the performance, which depends on the functions $\mathcal{J}_i(.)$, the rate-split $\mathbf{r}$, and the joint probability density function $p_\gamma(\rho_1, \dots, \rho_N)$ of the instantaneous SNRs. In the next section, we address the optimization of the performance $P_{ub}(\mathbf{r}, \gamma)$ for a $N = 2$ Rayleigh block fading channel with the Arikan's polarization kernel.

IV. Optimization for $N = 2$ blocks

We focus now on the case of the block fading channel with 2 blocks, which gives all the insights for a generalization to a larger number of fading blocks and polarization kernel sizes that will be given in an extended version of this paper.

A. Outage boundaries of the polarized block fading channel

Let us consider the $N = 2$ block fading channel. In Fig. 3, the outage boundaries defined by the set of points (ρ_1, ρ_2) such that $\mathcal{I}(\rho_1) + \mathcal{I}(\rho_2) = 2R$, are drawn for several values of R. The points bellow these boundaries are associated to outage events, the probability of outage being the integral of the joint distribution of (ρ_1, ρ_2) on the outage region Ω_{out}, where

$$\Omega_{out}(R) = \{(\rho_1, \rho_2) \in \mathbb{R}^2 | \mathcal{I}(\rho_1) + \mathcal{I}(\rho_2) < 2R\} \quad (5)$$

and

$$P_{out}(R, \gamma) = \oint_{\Omega_{out}(R)} p_\gamma(\rho_1, \rho_2) d\rho_1 d\rho_2. \quad (6)$$

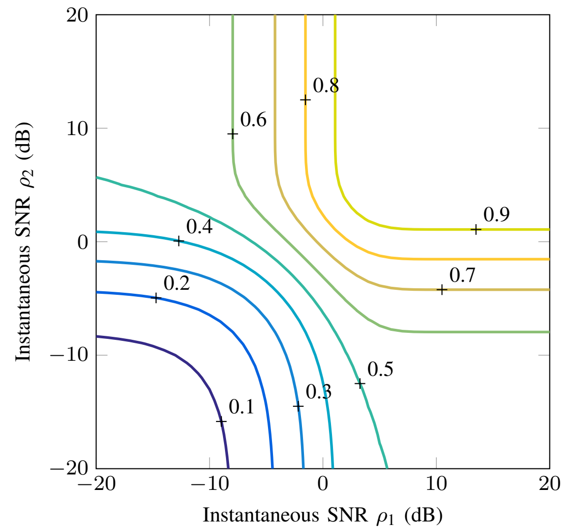

Fig. 3. Outage limit of the $N = 2$ block fading channel, as a function of the average rate R shown on the limit curves.

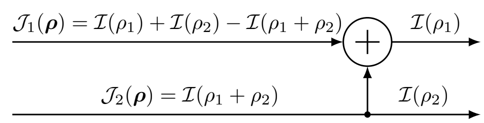

Fig. 4. Conditional mutual information for a polarization kernel of size 2.

As already mentioned in [6], the outage region $\Omega_{out}(R)$ is independent of the joint distribution of (ρ_1, ρ_2). The shapes of the outage boundaries illustrate the results obtained from the Singleton bound on block fading channels ([1] for error correcting codes, and [10] for the BPSK input mutual information) that the full diversity order is only observed for $R \leq 0.5$. The upper bound $P_{ub}(\mathbf{r}, \gamma)$ can be optimized, for a given long term SNR γ and joint probability density function $p(\rho_1, \rho_2)$, as

$$(\hat{r}_1, \hat{r}_2) = \arg \min_{r_1, r_2 | r_1 + r_2 = 2R} \oint_{\Omega_1(r_1) \bigcup \Omega_2(r_2)} p_\gamma(\rho_1, \rho_2) d\rho_1 d\rho_2 \quad (7)$$

where $\Omega_i(r_i) = \{\boldsymbol{\rho} \in \mathbb{R}^2 | \mathcal{J}_i(\boldsymbol{\rho}) < r_i\}$. Since the rate-split $(\hat{r}_1, \hat{r}_2)$ is determined semi-statically, i.e., for each long-term SNR value γ, it can be computed offline and stored for future use in the long-term link adaptation process.

Let us now consider the effect of the Arikan's polarization kernel of size 2 illustrated in Fig. 4. As stated in [6], for a given realization of the SNRs (ρ_1, ρ_2), the conditional mutual information functions are given by $\mathcal{J}_1(\boldsymbol{\rho}) = \mathcal{P}(\rho_1, \rho_2)$ and $\mathcal{J}_2(\boldsymbol{\rho}) = \mathcal{R}(\rho_1, \rho_2)$. The function

$$\mathcal{R}(\rho_1, \rho_2) = \mathcal{I}(\rho_1 + \rho_2) \quad (8)$$

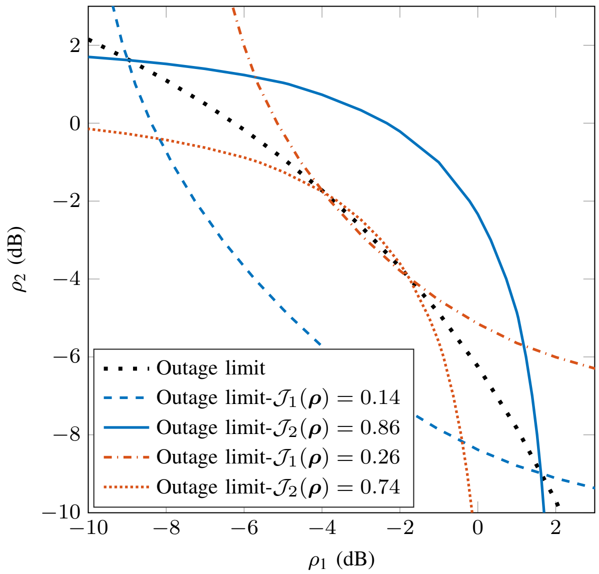

Fig. 5. Outage regions for 2 blocks, an average rate $R = 0.5$, $r_1 = 0.14$ and $r_2 = 0.86$, or $r_1 = 0.26$ and $r_2 = 0.74$.

is obtained when the input Z_1 of the first channel branch is known. Indeed, the LLR of Z_2 is obtained from a repetition check operation $L_1 + L_2$, where L_1 and L_2 are the LLRs associated to Y_1 and Y_2, respectively. Thus, the equivalent channel is an AWGN channel with instantaneous SNR $\rho_1 + \rho_2$ and inherent diversity order 2. The function

$$\mathcal{P}(\rho_1, \rho_2) = \mathcal{I}(\rho_1) + \mathcal{I}(\rho_2) - \mathcal{I}(\rho_1 + \rho_2) \tag{9}$$

is obtained when the input Z_2 of the second channel branch is unknown. Indeed, the LLR of Z_1 is obtained from a parity check operation $2\tanh^{-1}(\tanh(L_1/2)\tanh(L_2/2))$. Thus, the equivalent channel is not AWGN.

The conditional mutual informations $\mathcal{J}_1(\boldsymbol{\rho}) = \mathcal{P}(\rho_1, \rho_2)$ and $\mathcal{J}_2(\boldsymbol{\rho}) = \mathcal{R}(\rho_1, \rho_2)$ are nondecreasing component-wise functions which, thanks to Proposition 2, guarantees no outage probability degradation by the effect of Arikan's polarization kernel.

Fig. 5 illustrates the outage limits of the outage regions $\Omega_i(r_i)$ associated to the i-th parallel coding branch of the polarized block fading channel. The average rate is set to $R = 0.5$, and two rate-split are shown: $[r_1, r_2] = [0.14, 0.86]$ and $[r_1, r_2] = [0.26, 0.74]$. We can observe how the rate-split changes the domain of integration of the joint probability density function $p(\rho_1, \rho_2)$, defined as the union of the regions bellow the outage limits of $\mathcal{J}_1(\boldsymbol{\rho})$ and $\mathcal{J}_2(\boldsymbol{\rho})$. The optimization problem (7) can be solved numerically by varying $r_1 \in [0, \min(1, 2R)]$, $r_2 = \min(1, 2R) - r_1$, and performing a numerical integration or a Monte-Carlo simulation.

B. Approximation of the rate-split for a Rayleigh fading

The outage regions $\Omega_1(r_1)$ and $\Omega_2(r_2)$ are defined in the SNR domain as

$$\Omega_1(r_1) = \{\boldsymbol{\rho} \in \mathbb{R}^2 | \mathcal{P}(\rho_1, \rho_2) < r_1\} \tag{10}$$
$$\Omega_2(r_2) = \{\boldsymbol{\rho} \in \mathbb{R}^2 | \mathcal{R}(\rho_1, \rho_2) < r_2\} \tag{11}$$

Let us define the region $\Omega'(\rho'_1, \rho'_2) = \{\boldsymbol{\rho} \in \mathbb{R}^2 | \rho_1 + \rho_2 < \rho'_1 + \rho'_2\} \cup \{\boldsymbol{\rho} \in \mathbb{R}^2 | \min(\rho_1, \rho_2) < \min(\rho'_1, \rho'_2)\}$, such that $r_1 = \mathcal{P}(\rho'_1, \rho'_2)$, $r_2 = \mathcal{R}(\rho'_1, \rho'_2)$, and $r_1 + r_2 = 2R$. By using the nondecreasing component-wise properties of $\mathcal{P}(\rho_1, \rho_2)$ and $\mathcal{R}(\rho_1, \rho_2)$, one can show that $\Omega_1(r_1) \cup \Omega_2(r_2) \subset \Omega'(\rho'_1, \rho'_2)$, whcih will allow to define an upper bound of $P_{ub}(\mathbf{r}, \gamma)$. When independent Rayleigh fading is considered, the integration of the joint probability density function $p_\gamma(\rho_1, \rho_2) = \frac{1}{\gamma^2} e^{-\rho_1/\gamma} e^{-\rho_2/\gamma}$ on the region $\Omega'(\rho'_1, \rho'_2)$ leads to

$$P_{ub}(\mathbf{r}, \gamma) \le 1 - \left(1 + \frac{|\rho'_1 - \rho'_2|}{\gamma}\right) e^{-\frac{\rho'_1 + \rho'_2}{\gamma}}. \tag{12}$$

The minimum of this upper bound can be obtained numerically for all set of candidate tuples (ρ'_1, ρ'_2) such that $\mathcal{P}(\rho'_1, \rho'_2) + \mathcal{R}(\rho'_1, \rho'_2) = \mathcal{I}(\rho'_1) + \mathcal{I}(\rho'_2) = 2R$. This set is obtained by a variation of $\mathcal{I}(\rho'_1)$ between 0 and $\min(1, 2R)$, a computation of ρ'_1 from a tabulated function $\mathcal{I}^{-1}(.)$, and $\rho'_2 = \mathcal{I}^{-1}(\min(1, 2R) - \mathcal{I}(\rho'_1))$. The best tuple $(\hat{\rho}'_1, \hat{\rho}'_2)$ provides an estimate of $(\hat{r}_1 = \mathcal{P}(\hat{\rho}'_1, \hat{\rho}'_2), \hat{r}_2 = \mathcal{R}(\hat{\rho}'_1, \hat{\rho}'_2))$ with low computational complexity. Fig. 6 shows, as a function of the SNR, the optimized choice of rate-split $(\hat{r}_1, \hat{r}_2)$ obtained with a Monte-Carlo simulation, and the rate-split obtained by the approximation (12), which results in a tight lower-estimation of $\hat{r}_1$.

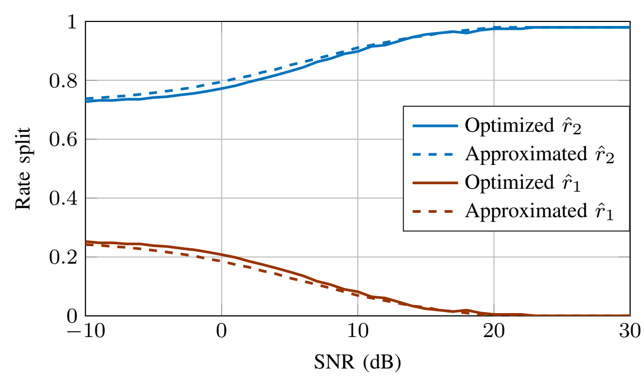

Fig. 6. Optimized rates $\hat{r}_1$ and $\hat{r}_2$ for an average rate $R = 0.49$, obtained by a Monte-Carlo simulation (solid) or by approximation (12) (dashed).

V. Polar codes as outer codes

In Proposition 1, we have shown that an optimized outage rate is achievable if each i-th component of the outer parallel code, shown in Fig. 2, is outage achieving for the equivalent BDMC with a quasi-static conditional mutual information $\mathcal{J}_i(.)$. It is now of interest to find a family of codes reaching this property.

Proposition 3. *If a polar code of rate r is capacity achieving on a BDMC with a LLR being non-decreasing with the channel parameter, it is outage achieving on the counterpart quasi-static BDMC. (Proof to be provided in the extended paper).*

The LLR associated to the observation Y'_j of a BPSK input AWGN channel with SNR ρ' is given by $4\rho' Y'_j$, and is non-decreasing in ρ'. Thus, according to Proposition 3, polar codes achieve the outage probability of quasi-static fading AWGN channels, whatever the probability density function of the fading.

Finally, when using polar codes as constituent outer parallel codes, the global code presented in Fig. 2 is itself a polar code. Thus, the decoder can jointly decode the global code with low complexity by using a serial cancellation or a belief propagation approach which asymptotically outperforms the achievable bound (4). In this case, the originality of our work lies in the fact that the frozen bits are not selected for the global code, but by an optimization of the sub-codes according to the rate-split design presented in this paper.

VI. Simulation results

The outage probabilities for an average rate $R = 0.49$ of the $N = 2$-block Rayleigh fading (BF) and the two repetitions Maximum Ratio Combining (MRC) channels are shown in Fig. 7, along with the upper bound performance (4) when the rate-split is taken according to Fig. 6. We first observe that the approximated rate-split does not lead to strong performance degradation. We observe that the upper bound of the performance is very close to the outage probability of the block fading channel for frame error rate above 10^{-1}, while the bound is loose for high SNR. Thus, the proposed scheme is expected to achieve good performance for systems working at frame error rates around 10^{-1}, such as with ARQ or HARQ retransmission schemes. Fig. 8 shows the result of the optimization of an ARQ-based transmission scheme without CSIT. For each SNR, the rate R is selected in order to maximize $R(1 - P_{out}(R, \gamma))$ for the outage-based performance, or $R(1 - P_{ub}(\mathbf{r}, \gamma))$ for the proposed scheme, where $\mathbf{r}$ is the best rate-split minimizing $P_{ub}(\mathbf{r}, \gamma)$ for a given SNR γ. We observe that, by using Arikan's kernel of size 2 as an inner code, polar codes can beat the throughput of the MRC-based repetition coding scheme and the parallel encoding scheme, and perform very close to the optimal for high throughput.

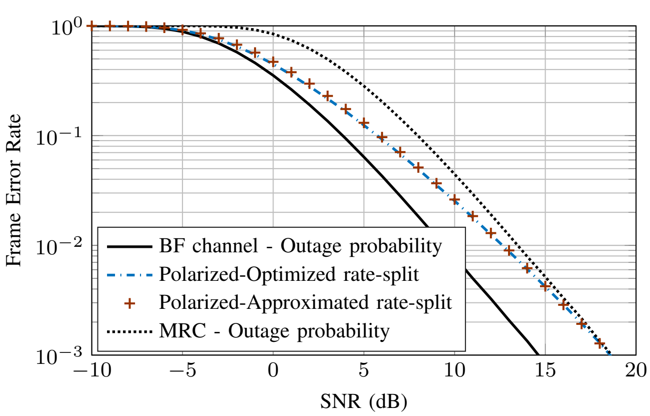

Fig. 7. Outage probabilities of the $N = 2$ block fading channel for an average rate $R = 0.49$, and upper bound performance of the polarized channel with an optimized and approximated rate-split.

VII. Conclusion

In this paper, we have proposed a statistical optimization of polar codes for block fading channels without CSIT. The polar codes are decomposed as the concatenation of an inner code defined by a Polarization Kernel and a set of outer parallel codes. The inner code allows to change the nature of the block fading channel into quasi-static parallel BDMCs which parameters are non-identically distributed and non-independent. The independent decoding of the parallel outer codes leads to a performance degradation. It can be limited by a statistical rate-splitting optimization which involves that the global polar code frozen bits are designed to be statistically robust to the block fading channel without CSIT. This scheme shows promising results for high throughput under ARQ. The future work will present the extension to a higher number of fading blocks.

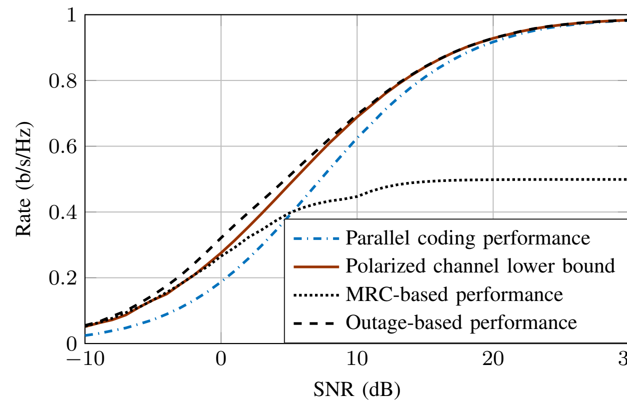

Fig. 8. Throughput of an ARQ scheme over a $N = 2$ block fading channel with parallel coding and concatenation to an outer polarization kernel.

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