

Coding for the Non-Orthogonal Amplify-and-Forward Cooperative Channel

Ghassan M. Kraidy, *Member, IEEE*, Nicolas Gresset, *Member, IEEE*, and Joseph J. Boutros, *Member, IEEE*

Abstract—In this work, we consider the problem of coding for the half-duplex non-orthogonal amplify-and-forward (NAF) cooperative channel where the transmitter to relay and the inter-relay links are highly reliable. We derive bounds on the diversity order of the NAF protocol that are achieved by a distributed space-time bit-interleaved coded modulation (D-ST-BICM) scheme under iterative APP detection and decoding. These bounds lead to the design of space-time precoders that ensure maximum diversity order and high coding gains. The word error rate performance of D-ST-BICM are also compared to outage probability limits.

I. INTRODUCTION

Signals transmitted over wireless channels undergo severe degradations due to effects such as path loss, shadowing, fading, and interference from other transmitters, in addition to thermal noise at the receiver. One major way to combat static fading is to provide diversity in either time, frequency, or space [1]. For this purpose, multiple-antenna systems that provide high orders of spatial diversity and high capacity have been extensively studied [2]. However, due to limited terminal sizes, the implementation of two or more antennas may be impossible. Based on the seminal works in [3] and [4], the authors in [5][6] set up a framework for *cooperative communications*, where multiple terminals use the resources of each other to form a virtual antenna array. Following these works, many researchers have proposed distributed communication schemes and analyzed their outage probability behavior such as in [7][8][9][10][11]. The main protocols that have been proposed are the *amplify-and-forward*, where the relay only amplifies the signal received from the source, before transmitting it to the destination, and the *decode-and-forward*, where the relay decodes the received signal before transmitting it to the destination. In this paper, we study the performance of distributed space-time bit-interleaved coded modulations (D-ST-BICM) schemes for non-orthogonal amplify-and-forward protocols. Furthermore, we focus on situations where the transmitter to relay and inter-relay links quality is highly better than the transmitter to receiver link quality. This situation

G.M. Kraidy was with CEA-LETI, MINATEC, 17 rue des martyrs, 38054, Grenoble, France. He is now with the Department of Electronics and Telecommunications, Norwegian University of Science and Technology (NTNU), O. S. Bragstads plass 2B, 7491 Trondheim, Norway (email: kraidy@ieee.org).

N. Gresset is with Mitsubishi Electric R&D Centre Europe, 1 allée de Beaulieu, Rennes, France (email: gresset@ieee.org).

J.J. Boutros is with Texas A&M University, Education City, Doha, Qatar (email: boutros@tamu.edu).

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occurs for example when deploying professional relays on top of buildings in a way to improve the link reliability in low coverage zones of a multi-cellular system.

The paper is organized as follows: Section II defines the Matryoshka block-fading channel, a channel that characterizes the cooperative protocol considered in this paper. In Section III, we describe the system model and all the parameters involved in our study. We then derive bounds on the diversity of D-ST-BICM for the minimum cooperation frame length in Section IV, and Section V extends these results for any length. Section VI shows simulation results for different network topologies, while Section VII gives the concluding remarks.

II. MATRYOSHKA BLOCK-FADING CHANNELS

In this paper, we consider the block-fading channel model in which a D-ST-BICM codeword undergoes a limited number of fading channel realizations, namely one fading coefficient per spatial path. For the sake of analysis, we define a block-fading channel model where the set of random variables of a higher diversity block always includes the set of random variables of a lower diversity block, in a way similar to nested Matryoshka dolls. This model was first introduced in [12].

Definition 1: Let us consider λ independent fading random variables (h_1, \dots, h_λ) providing a total diversity order of λ . Let $\mathcal{M}(\mathcal{D}, \mathcal{L})$ be a channel built from the concatenation of $|\mathcal{D}|$ blocks, where $\mathcal{D} = \{\mathcal{D}_i\}_i$ and $\mathcal{L} = \{\mathcal{L}_i\}_i$ are respectively the sets of diversity orders and lengths of each block. As usual, the integer $|\chi|$ denotes the cardinality of the set χ . The i -th block has a diversity order equal to \mathcal{D}_i and its fading set is $\mathcal{S}(i)$ with $|\mathcal{S}(i)| = \mathcal{D}_i$, $\mathcal{D}_i \leq \lambda$ fading random variables, such that $\mathcal{S}(i) \subset \mathcal{S}(i-1)$. Thus, we have $\forall i > j, \mathcal{D}_i \leq \mathcal{D}_j$ and $\mathcal{S}(1) = \{h_1, h_2, \dots, h_\lambda\}$ or equivalently $\mathcal{D}_1 = \lambda$ is the maximum diversity order. This channel defined by nested fading sets is referred to as a Matryoshka channel and it is illustrated in Fig. 1.

Let us now transmit a BPSK-modulated and interleaved codeword of a rate- R_c code over the $\mathcal{M}(\mathcal{D}, \mathcal{L})$ channel. First, let us focus on the pairwise error probability (PEP) of two given binary codewords c and c' . Due to the channel model, the diversity order of this PEP is equal to the diversity order of the lowest index block observing a non-zero part of $c - c'$. The performance of the coded modulation has a diversity order upper-bounded by δ_{max} defined as follows:

Proposition 1: The diversity observed after decoding a rate- R_c linear code transmitted over a $\mathcal{M}(\mathcal{D}, \mathcal{L})$ channel is upper-bounded by $\delta_{max} = \mathcal{D}_i$ where i is given by the following

\mathcal{D}_1	\mathcal{D}_2	$\mathcal{D}(\mathcal{D})$
$\mathcal{S}(1) = \{\alpha_1, \dots, \alpha_\lambda\}$	$\mathcal{S}(2) \subset \mathcal{S}(1)$		$\mathcal{S}(\mathcal{D}) \subset \mathcal{S}(\mathcal{D} - 1)$
$\leftarrow \mathcal{L}_1 \text{ bits} \rightarrow$	$\leftarrow \mathcal{L}_2 \text{ bits} \rightarrow$		$\leftarrow \mathcal{L}(\mathcal{D}) \text{ bits} \rightarrow$

Fig. 1. Definition of a Matryoshka block-fading channel with $|\mathcal{D}|$ nested fading sets.

inequalities:

$$\sum_{k=1}^{i-1} \mathcal{L}_k < R_c \sum_{k=1}^{|\mathcal{D}|} \mathcal{L}_k \leq \sum_{k=1}^i \mathcal{L}_k \quad (1)$$

and δ_{max} is achievable for any systematic linear code.

Proof: This proof is inspired from the Singleton bound's one. The code C has length N and dimension K , where $N = \sum_{k=1}^{|\mathcal{D}|} \mathcal{L}_k$ and $K = R_c N$. If $K > \sum_{k=1}^{i-1} \mathcal{L}_k$, whatever the code is, a puncturing of the last $\sum_{k=i}^{|\mathcal{D}|} \mathcal{L}_k$ bits leads to a zero minimum Hamming distance because $\sum_{k=i}^{|\mathcal{D}|} \mathcal{L}_k > N - K$. This means that there always exists two codewords c and c' such that the last $\sum_{k=i}^{|\mathcal{D}|} \mathcal{L}_k$ bits of $c - c'$ are null, and involves that $\delta_{max} \leq \mathcal{D}_i$.

Let us now suppose that the code is linear and systematic. If the information bits are transmitted on the blocks of highest diversity order and if $K \leq \sum_{k=1}^i \mathcal{L}_k$, the Hamming distance after puncturing the last $\sum_{k=i+1}^{|\mathcal{D}|} \mathcal{L}_k$ bits remains strictly positive and induces that $\delta_{max} \geq \mathcal{D}_i$. \square

It is straightforward to show that the bound on the diversity order applies to any discrete modulation.

As a remark, in order to achieve the upper-bound on the diversity of a block-fading channel, non-zero bits of word $c - c'$ should be placed in as many independent blocks as given by the Singleton bound. For Matryoshka channels, the bound is achieved as soon as one non-zero bit of any word $c - c'$ is placed in a block of diversity higher than δ_{max} .

III. SYSTEM MODEL AND PARAMETERS

We consider the cooperative amplify-and-forward fading channel, where terminals have a single antenna. We impose the half-duplex constraint, in which terminals cannot transmit and receive signals simultaneously. We consider the TDMA-based Protocol I from [10] that is also known as the non-orthogonal amplify-and-forward (NAF) protocol. For cases with more than one relay, we consider the M -slot β -relay sequential slotted amplify-and-forward (SSAF) cooperative protocol [13], where inter-relay communication is allowed as illustrated in Fig. 2. The source transmits in all time slots, and starting from the second slot, only one relay scales and transmits the message received in the previous time slot. The reason we use this protocol is that it outperforms the classical β -relay NAF protocol in terms of outage probability [14]. This protocol gives the following signal model:

$$y_{d_i} = \sqrt{\mathcal{E}_i} h_{sd} x_i + \sqrt{1 - \mathcal{E}_i} \hat{h}_{r_{i-1}d} \gamma_{i-1} y_{r_{i-1}} + w_{d_i} \quad (2)$$

$$y_{r_i} = \sqrt{\mathcal{E}_i} \hat{h}_{sr_i} x_i + \sqrt{1 - \mathcal{E}_i} \hat{h}_{r_{i-1}r_i} \gamma_{i-1} y_{r_{i-1}} + w_{r_i} \quad (3)$$

with $i = 1, \dots, M$. We have that y_{r_0} , h_{r_0d} , and γ_0 are null. Subscripts s , d , and r_i correspond to *source*, *destination*, and

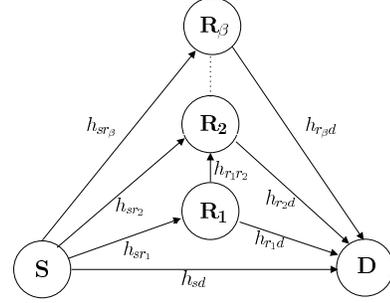


Fig. 2. Slotted amplify-and-forward protocol channel model

i -th effective relay [13]. The unit variance complex symbol x_i is transmitted in the i -th slot, the received signal at the destination in the i -th time slot is y_{d_i} , while y_{r_i} is the signal received by the i -th effective relay. The coefficients \mathcal{E}_i represent the energy transmitted by the source in the i -th slot. The \hat{h}_{uv} are the complex Gaussian fading coefficients given by:

$$\begin{aligned} \hat{h}_{sr_i} &= h_{sr_j}, & j &= [(i-1) \bmod (\beta)] + 1 \\ \hat{h}_{r_i d} &= h_{r_j d}, & j &= [(i-1) \bmod (\beta)] + 1 \\ \hat{h}_{r_i r_k} &= h_{r_j r_\ell}, & j &= [(i-1) \bmod (\beta)] + 1, \\ & & \ell &= [(k-1) \bmod (\beta)] + 1 \end{aligned}$$

The h_{uv} coefficients are the fading coefficients between devices u and v . The w_{d_i} and w_{r_i} are additive white Gaussian noise (AWGN) components. The γ_i are the energy normalization coefficients at the i -th relay, subject to $\mathbb{E}|\gamma_i y_{r_i}|^2 \leq 1$, and $\gamma_0 = 0$. In matrix form, the channel model becomes:

$$\mathbf{y}_d = \mathbf{x}\mathbf{H} + \mathbf{w}_c = \mathbf{z}\mathbf{S}\mathbf{H} + \mathbf{w}_c \quad (4)$$

where \mathbf{y}_d is the length- M vector of received signals and \mathbf{z} is the length- M vector of 2^m -QAM symbols. \mathbf{S} is a $M \times M$ precoding matrix, and \mathbf{H} is upper-triangular as shown in (5). Finally, the vector \mathbf{w}_c is a length- M colored Gaussian noise vector as given by (6). We set:

$$\mathbf{\Gamma} = \mathbf{E}[\mathbf{w}_c \mathbf{w}_c^\dagger] = 2N_0 \mathbf{\Theta} \quad (7)$$

Where the \dagger operator denotes transpose conjugate. By performing a Cholesky decomposition on $\mathbf{\Theta}$, we get:

$$\mathbf{\Theta} = \mathbf{\Psi}^\dagger \mathbf{\Psi} \quad (8)$$

Thus the equivalent channel model becomes:

$$\mathbf{y}_d \mathbf{\Psi}^{-1} = \mathbf{z}\mathbf{S}\mathbf{H}\mathbf{\Psi}^{-1} + \mathbf{w} \quad (9)$$

where \mathbf{w} is a white Gaussian noise vector. Digital transmission is made as follows: Uniformly distributed

$$\mathbf{H} = \begin{bmatrix} \sqrt{\mathcal{E}_1}h_{sd} & \sqrt{\mathcal{E}_1(1-\mathcal{E}_2)}\gamma_1\hat{h}_{sr_1}\hat{h}_{r_1d} & \sqrt{\mathcal{E}_1(1-\mathcal{E}_2)(1-\mathcal{E}_3)}\gamma_1\gamma_2\hat{h}_{sr_1}\hat{h}_{r_1r_2}\hat{h}_{r_2d} & \cdots \\ 0 & \sqrt{\mathcal{E}_2}h_{sd} & \sqrt{\mathcal{E}_2(1-\mathcal{E}_3)}\gamma_2\hat{h}_{sr_2}\hat{h}_{r_2d} & \cdots \\ 0 & 0 & \sqrt{\mathcal{E}_3}h_{sd} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix} \quad (4)$$

$$\mathbf{w}_c = [w_1 \quad w_2 \quad w_3 \quad \cdots] \quad (5)$$

with:

$$\begin{aligned} w_1 &= w_{d,1} \\ w_2 &= \sqrt{(1-\mathcal{E}_2)}\gamma_1\hat{h}_{r_1d}w_{r,1} + w_{d,2} \\ w_3 &= \sqrt{(1-\mathcal{E}_2)(1-\mathcal{E}_3)}\gamma_1\gamma_2\hat{h}_{r_1r_2}\hat{h}_{r_2d}w_{r,1} + \sqrt{(1-\mathcal{E}_3)}\gamma_2\hat{h}_{r_2d}w_{r,2} + w_{d,3} \end{aligned}$$

information bits are fed to a binary convolutional encoder. Coded bits $\{c_i\}$ are then interleaved and Gray mapped into QAM symbols. The QAM symbols are then rotated via \mathbf{S} and transmitted on the SSAF channel defined by \mathbf{H} given in (4). The coherent detector at the destination computes an extrinsic information $\xi(c_i)$ based on the knowledge of \mathbf{H} , the received vector \mathbf{y}_d , and independent *a priori* information $\pi(c_j)$ for all coded bits. The channel decoder then computes *a posteriori* probabilities (APP) based on the de-interleaved extrinsic information coming from the detector using the forward-backward algorithm [15]. The transmitted information rate is equal to $R = R_c m$ bits per channel use, where the cardinality of the QAM constellation is 2^m .

As a remark, one precoded symbol at the output of \mathbf{S} is transmitted over a row of the channel matrix \mathbf{H} and thus experiences a set of random variables $\{h_{sd}, h_{sr_i}h_{r_id}, \dots, h_{sr_i}h_{r_i r_{i+1}} \cdots h_{r_\beta d}\}$. If we assume that the quality of the source to relays and inter-relays links is much better than the source to destination or relay to destination links, we can then focus on the h_{sd} or h_{r_id} random variables to understand the diversity behavior of such a system. Indeed, in the context of professional relay deployment on top of buildings, we may assume that the relays are placed and have their antennas tuned to ensure a good link quality with the base station. Furthermore, in the case of detect-and-forward or decode-and-forward protocols, this assumption is still relevant. Finally, one precoded symbol transmitted on the i -th row of the channel matrix sees a set of $\beta + 2 - i$ fading variables included in the set seen by a symbol sent on the $i - 1$ -th row. Hence, we will see in the sequel that the equivalent channels obtained by the use of a sequential slotted amplify and forward protocol fall into the class of Matryoshka channels.

IV. THE DIVERSITY OF D-ST-BICM OVER $\beta + 1$ -SLOT SSAF CHANNELS

The maximum diversity inherent to the SSAF channel is $d_{max} = \beta + 1$, and it can be collected by an APP detector (at the destination) if a full-diversity linear precoder is used at the transmitter. The precoder mixes the $\beta + 1$ constellation symbols being transmitted on the channel providing full diversity with

uncoded systems and without increasing the complexity at the detector. Using precoders that process spreading among more than $\beta + 1$ time slots can further improve the performance. From an algebraic point of view, a linear precoder of size $(\beta + 1)^2 \times (\beta + 1)^2$ is the optimal configuration to achieve good coding gains (without channel coding) [11] at the price of an increase in detection complexity (the complexity of an exhaustive APP detector grows exponentially with the number of dimensions).

On the other hand, for coded modulations transmitted on block-fading channels, the channel decoder is capable of collecting a certain amount of diversity that is however limited by the Singleton bound [16]. In [17][19][18], the modified Singleton bound taking into account the rotation size over a MIMO block-fading channel is used to achieve the best tradeoff between complexity and diversity. For this purpose, we derive hereafter an upper-bound on the diversity order of a coded transmission over a precoded $\beta + 1$ -slot SSAF channel, and then deduce the precoding strategy to follow in order to achieve full diversity.

A. Precoded $\beta + 1$ -slot SSAF channel models and associated bounds

1) *Non-precoded $\beta + 1$ -slot SSAF channels with equal per-slot spectral efficiency:* We will first assume that the interleaver of the BICM is ideal, which means that for any pair of codewords (c, c') , the ω non-zero bits of $c - c'$ are transmitted in different blocks of $\beta + 1$ time periods, which means that no inter-slot inter-bit interference is experienced. The interleaving, modulation and transmission through the channel convert the codewords c and c' onto points \mathcal{C} and \mathcal{C}' in a Euclidean space. For a fixed channel, the performance is directly linked to the Euclidean squared distance $|\mathcal{C} - \mathcal{C}'|^2$, that can be rewritten as a sum of ω squared Euclidean distances associated to the ω non-zero bits of $c - c'$. For each of the ω squared Euclidean distances, we can build an equivalent channel model which corresponds to the transmission of a BPSK modulation over one row of the channel matrix \mathbf{H} . Thus, several squared Euclidean distances appear to be transmitted on the same equivalent channel and the squared distance

$|\mathcal{C} - \mathcal{C}'|^2$ can be factorized as follows: $|\mathcal{C} - \mathcal{C}'|^2 = \sum_{i=1}^{\beta+1} d_i^2$ where d_i^2 is linearly dependent on the norm of the k -th row of \mathbf{H} .

In other words, at the output of the APP detector, an equivalent block-fading channel is observed and the constituent blocks do not have the same intrinsic diversity order: A soft output belonging to the j -th block carries the attenuation coefficients $\{h_{sd}; h_{r_j d}; \dots; h_{r_{\beta} d}\}$. As a remark, blocks are sorted such that the j -th block carries a diversity order of $\beta + 2 - j$ and the subset of realizations of random variables observed in the i -th row of \mathbf{H} is included in the subset of random variables observed in the $i - 1$ -th row of \mathbf{H} . As the same modulation is used on each time slot of the relaying protocol, each block length is equal to $N/(\beta + 1)$.

Finally, the equivalent $\beta + 1$ -slot SSAF channel at the output of the APP detector is a Matryoshka $\mathcal{M}([\beta + 1, \beta, \dots, 1], [N/(\beta + 1), \dots, N/(\beta + 1)])$ channel, where N is the number of coded bits per codeword. With this observation, we can conclude that the upper-bound on the diversity order of a non-precoded SSAF channel is

$$\delta_{max,1}(\beta, R_c) = 1 + \lfloor (1 - R_c)(\beta + 1) \rfloor \quad (10)$$

which is equal to the classical Singleton bound on the diversity order of block-fading channels [16], with the difference that it can be achieved by any systematic code.

2) *Non-precoded $\beta + 1$ -slot SSAF channels with unequal per-slot spectral efficiency*: For the sake of generalization, we now suppose that modulations with different spectral efficiencies are sent over the $\beta + 1$ slots of the cooperation frame. We define m_k as the number of bits carried by one symbol of the modulation transmitted on the k -th time slot. In this case, the block fading channel is a $\mathcal{M}([\beta + 1, \beta, \dots, 1], [\frac{m_1 N}{\sum_{k=1}^{\beta+1} m_k}, \frac{m_2 N}{\sum_{k=1}^{\beta+1} m_k}, \dots, \frac{m_{\beta+1} N}{\sum_{k=1}^{\beta+1} m_k}])$ Matryoshka channel. By applying (1), we obtain that if:

$$R_c \leq \frac{\sum_{j=1}^i m_j}{\sum_{k=1}^{\beta+1} m_k} \quad (11)$$

then the achievable diversity order is $d = \beta + 2 - i$.

For a given distribution of spectral efficiencies, it is better to choose $m_1 > m_2 > \dots > m_{\beta+1}$, as a higher diversity order might be achieved for a given coding rate. It is also clear that higher coding rates than in (10) can be attained for a given target diversity. However, the bound on the diversity does not give any information on the coding gain of the coded scheme. We will see later that a fine tuning of the choice of the spectral efficiencies might be needed to optimize the coding gain. For example, the orthogonal amplify-and-forward protocol leads to $m_{k>1} = 0$, which provides full diversity whatever the code rate is but exhibits a poor coding gain [7].

3) *Precoded $\beta + 1$ -slot SSAF channels with equal per-slot spectral efficiency*: Let us now introduce a linear precoder that rotates symbols of s different diversity blocks together. First of all, let us focus on two different scenarios:

- The linear precoder size is lower than (or equal to) $\beta + 1$. In this case, the dimension of the received vector \mathbf{y}_d remains unchanged, thus there is no increase in detection complexity when an exhaustive APP detector is used, and

no delay is introduced to the protocol. The authors in [20] considered the design of such precoders for uncoded systems.

- The linear precoder size is lower than (or equal to) $(d+1)(\beta+1) \times (d+1)(\beta+1)$, where d is the delay (i.e. the source broadcasts for $d + 1$ time slots before the relays start to cooperate). In this case, the complexity of the detector increases exponentially with d . As mentioned previously, these precoders are mandatory to achieve optimal performance for uncoded systems. As we focus on channel coding issues in this work, delay-precoders will not be considered in the sequel.

We will now present two precoding strategies and compute the bound (1) for these two particular cases.

a) *First strategy: a single precoder*: First, let us assume that s diversity blocks of size $N/(\beta + 1)$ are linearly precoded together, then the diversity order of the new $sN/(\beta + 1)$ -length block is the maximum diversity order of the precoded blocks. As the other blocks keep their own diversity, it seems natural to maximize their diversity orders in a way to increase the coding gain at the output of the decoder (The best performance is achieved for a block-fading channel with diversity orders as equal as possible.). The length of the precoder input vector is $\beta + 1$. We propose to precode the first block with the $s - 1$ last blocks, i.e. the highest diversity order with the $s - 1$ lowest ones. At the output of the APP detector, the channel model is a Matryoshka $\mathcal{M}(\mathcal{D}, \mathcal{L})$ channel where $\mathcal{D} = [\beta + 1, \beta, \dots, s]$ and $\mathcal{L} = [sN/(\beta + 1), N/(\beta + 1), \dots, N/(\beta + 1)]$, which leads to the following upper-bound on the diversity order:

$$\delta_{max,2}(\beta, R_c, s) = \min(s + \lfloor (1 - R_c)(\beta + 1) \rfloor, \beta + 1) \quad (12)$$

Indeed, by replacing $\mathcal{D} = [\beta + 1, \beta, \dots, s]$ and $\mathcal{L} = [sN/(\beta + 1), N/(\beta + 1), \dots, N/(\beta + 1)]$ in (1), we observe that if $R_c \leq s/(\beta + 1)$ then $i = 1$ and $\delta_{max,2}(\beta, R_c, s) = \mathcal{D}_1 = \beta + 1$. Else, if $R_c > s/(\beta + 1)$, then $s + i - 1 = \lfloor R_c(\beta + 1) \rfloor$ which leads to $\delta_{max,2}(\beta, R_c, s) = \beta + 2 - i = s + \lfloor (1 - R_c)(\beta + 1) \rfloor$. Note that, in the representation of Fig. 1, we have that $|\mathcal{D}| = \lambda - s + 1$ in this case. If $s = 1$, then $\delta_{max,2}(\beta, R_c, s)$ is equal to the Singleton bound on the diversity order of an uncorrelated block-fading channel with equal per-block diversity. If $s \geq 1$, $\delta_{max,2}(\beta, R_c, s)$ is greater than the upper-bound on the diversity order for block-fading channels. For example, the full diversity order cannot be achieved for the transmission of a $s = 2$ -precoded BICM with rate $2/3$ on a block-fading channel with diversity order 3 (the diversity is upper-bounded by 2). For the SSAF channel, the full diversity order can be achieved in that case, as shown in Fig. 3. Fig. 4 and 5 show the values of $\delta_{max,2}(\beta, R_c, s)$ for different coding rates with respect to the number of relays and the value of s . We can notice that full diversity is obtained with $s \geq (\beta + 1)R_c$ in all configurations.

b) *Second strategy: $(\beta + 1)/s$ precoders*: Let us assume that s divides $\beta + 1$, we can then use $(\beta + 1)/s$ precoders: The first precodes the highest diversity order block with the $s - 1$ lowest ones. The second, if any, precodes the second highest diversity order block with the $s - 1$ lowest non-precoded ones, and so on. By using this precoding strategy that includes several independent precoders, we further increase

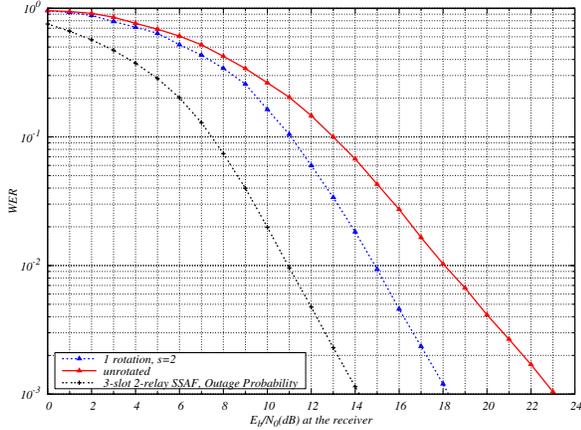


Fig. 3. Two-relay SAF cooperative channel, $R_c=2/3$ RSC (25,37,35)₈ code, BPSK modulation, $N = 1440$.

$\beta \setminus s$	1	2	3	4	5
1	2	2			
2	2	3	3		
3	3	4	4	4	
4	3	4	5	5	5
5	4	5	6	6	6
6	4	5	6	7	7
7	5	6	7	8	8
8	5	6	7	8	9

Fig. 4. $\delta_{max,2}(\beta, R_c, s)$ for $R_c = 1/2$

$\beta \setminus s$	1	2	3	4	5	6
1	1	2				
2	1	2	3			
3	2	3	4			
4	2	3	4	5		
5	2	3	4	5	6	
6	2	3	4	5	6	7
7	3	4	5	6	7	8

Fig. 5. $\delta_{max,2}(\beta, R_c, s)$ for $R_c = 3/4$

the diversity of the extrinsic probabilities at the input of the decoder, and consequently the diversity at the output of the decoder. Indeed, the equivalent $\mathcal{M}(\mathcal{D}, \mathcal{L})$ channel has parameters $\mathcal{D} = [\beta + 1, \beta, \dots, \beta + 2 - (\beta + 1)/s]$ and $\mathcal{L} = [sN/(\beta + 1), \dots, sN/(\beta + 1)]$, which leads to the following upper-bound on the diversity order:

$$\delta_{max,3}(\beta, R_c, s) = \min \left(\frac{(\beta+1)(s-1)}{s} + 1 + \left\lfloor \frac{(1-R_c)(\beta+1)}{s} \right\rfloor, \beta + 1 \right) \quad (13)$$

It can be easily shown that

$$\delta_{max,2}(\beta, R_c, s) \leq \delta_{max,3}(\beta, R_c, s) \quad (14)$$

However, the maximum diversity order $\delta_{max,2}(\beta, R_c, s) = \delta_{max,3}(\beta, R_c, s) = \beta + 1$ is achieved for the same $s \geq (\beta + 1)R_c$. The advantage of $\delta_{max,3}(\beta, R_c, s)$ over $\delta_{max,2}(\beta, R_c, s)$ is for non-full diversity schemes. In addition, it is important to note that the bounds in (12) and (13) have straight-forward applications to systems employing delay precoders.

4) *Precoded $\beta+1$ -slot SSAF channels with unequal per-slot spectral efficiencies:* Now we reconsider the scenario of Section IV-A.2, in which different modulation sizes are sent over the blocks. In addition, we consider that a space-time precoder with spreading s combines the symbol having maximum diversity with those having the least $s - 1$ diversity orders. We thus obtain a $\mathcal{M}(\mathcal{D}, \mathcal{L})$ Matryoshka channel with $\mathcal{D} [\beta + 1, \beta, \dots, s]$ and $\mathcal{L} = \left[\frac{(m_1 + \sum_{\gamma=1}^{s-1} m_{\beta+2-\gamma})N}{\sum_{k=1}^{\beta+1} m_k}, \frac{m_2 N}{\sum_{k=1}^{\beta+1} m_k}, \dots, \frac{m_{\beta+2-s} N}{\sum_{k=1}^{\beta+1} m_k} \right]$. By applying (1), we obtain that if:

$$R_c \leq \frac{\sum_{j=1}^i m'_j}{\sum_{k=1}^{\beta+1} m_k} \quad (15)$$

with:

$$m'_1 = m_1 + \sum_{\gamma=1}^{s-1} m_{\beta+2-\gamma}$$

$$m'_j = m_j \text{ otherwise}$$

then the achievable diversity order is $d = \beta + 2 - i$.

Thus, the parameters s, m_1, \dots, m_2 allow for a fine tuning of the target diversity for a given coding rate. This tuning allows to further improve the coding gain. Unfortunately, the theoretical analysis of coding gains for coded modulations on block fading channels is difficult and often solved by extensive computer simulations. Hence, the analysis of such a design is out of the scope of this paper, which mainly focuses on diversity orders optimization.

V. THE DIVERSITY OF D-ST-BICM OVER M -SLOT SSAF CHANNELS ($M > \beta + 1$)

So far, we have considered the β -relay SSAF protocol with length- $\beta + 1$ cooperation frames. In [13], the authors consider a cooperation scheme (for 2-relay SSAF and higher) in which the cooperation frame is stretched in a way to protect more symbols. In other words, we consider the M -slot β -relay SSAF protocol with:

$$M = \beta + 1 + \alpha \quad (16)$$

where α is the number of additional slots. The goal of this extension is to increase the number of coded bits that experience full diversity. The first $1 + \alpha$ symbols in \mathbf{x} from (4) will have maximum diversity, which reduces to the first symbol having maximum diversity in the $\beta + 1$ -slot SSAF scenario. However, this additional protection entails an increase in the size of \mathbf{x} , thus complexity at the APP detector increases as well.

An illustration of this scheme is provided in Fig. 6 for the 7-slot 2-relay SSAF protocol; the source always transmits a constellation symbol, and starting from the second time slot, the relays cooperate in a round robin way; in this case, the first 5 out of a total of 7 constellation symbols have a maximum diversity $d_{max} = \beta + 1 = 3$. It is then clear that this protocol allows to achieve full diversity with higher coding rates. In the sequel we will provide bounds on the diversity order of coded modulations under this cooperative protocol.

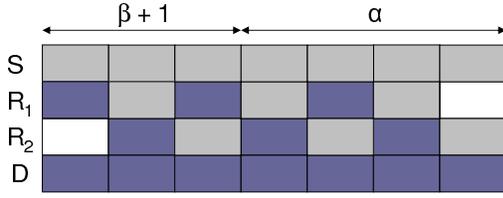


Fig. 6. Example of a 7-slot SSAF protocol with two relays. The cooperation frame has length $M = \beta + 1 + \alpha = 7$, a light gray rectangle means that the terminal is emitting, a dark blue rectangle means the terminal is receiving. A white rectangle means the terminal is inactive.

A. Non-precoded M -slot SSAF

We first consider the $(\beta + 1 + \alpha)$ -slot β -relay SSAF protocol without precoding. We thus obtain a Matryoshka block-fading channel as $\mathcal{M}(\mathcal{D}, \mathcal{L})$ with $\mathcal{D} = [\beta + 1, \beta, \dots, 1]$ and $\mathcal{L} = [(1 + \alpha)N/(\beta + 1 + \alpha), N/(\beta + 1 + \alpha), \dots, N/(\beta + 1 + \alpha)]$. This means that in a cooperation frame of length $(\beta + 1 + \alpha)$, there are $1 + \alpha$ symbols that have maximum diversity $d_{max} = \beta + 1$. This makes clear the fact that higher coding rates can be attained with this scheme. The diversity of a non-precoded BICM over this protocol is given by:

$$\delta_{max,4} = \min(1 + \lfloor (\beta + 1 + \alpha)(1 - R_c) \rfloor, \beta + 1), \quad \beta \geq 2 \quad (17)$$

Hence, we attain the maximum diversity order if:

$$R_c \leq \frac{1 + \alpha}{\beta + 1 + \alpha} \quad (18)$$

which implies that we can - theoretically - achieve full diversity with a coding rate getting close to 1 as α increases, but at the price of an APP detection complexity increase. To

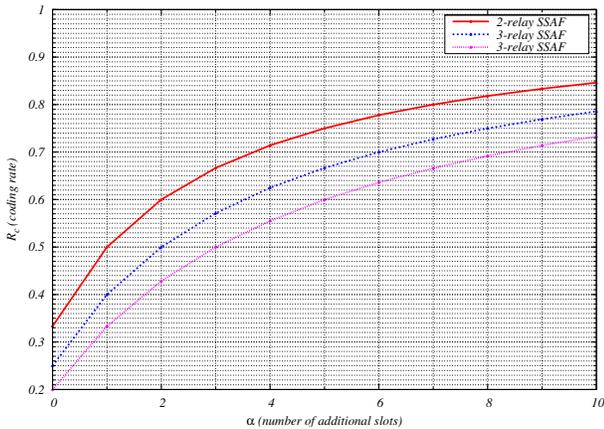


Fig. 7. Maximum coding rates R_c that can achieve $d_{max} = \beta + 1$ over the $(\beta + 1 + \alpha)$ -slot β -relay SSAF Matryoshka channel.

illustrate this bound on diversity, Fig. 7 gives examples of the 2-relay, 3-relay, and 4-relay SSAF channels. We noticed that with increasing the number of slots, the maximum coding rate has a logarithmic-like growth, while the complexity at the detector increases exponentially (as the cardinality of the received vector \mathbf{y}_d is $2^{(\beta+1+\alpha)m}$). This means that only few additional slots can be practically added to the cooperation frame in order to provide a reasonable rate / complexity tradeoff.

B. Precoded M -slot SSAF

If we precode the first symbol with maximum diversity with the $s - 1$ symbols having the lowest diversity orders we obtain a $\mathcal{M}(\mathcal{D}, \mathcal{L})$ block-fading channel where $\mathcal{D} = [\beta + 1, \beta, \dots, 1]$ and $\mathcal{L} = [(s + \alpha)N/(\beta + 1 + \alpha), N/(\beta + 1 + \alpha), \dots, N/(\beta + 1 + \alpha)]$. It is clear then that we provide $s + \alpha$ symbols having maximum diversity with precoding. The bound on diversity with a single precoder is given by:

$$\delta_{max,5} = \min(s + \lfloor (\beta + 1 + \alpha)(1 - R_c) \rfloor, \beta + 1), \quad \beta \geq 2 \quad (19)$$

Full diversity is obtained for

$$R_c \leq \frac{s + \alpha}{\beta + 1 + \alpha} \quad (20)$$

which, again, shows that linear precoding can be used to increase the obtained diversity without increasing the complexity of an optimal APP detector.

VI. SIMULATION RESULTS

In this section, word error rate performances of different D-ST-BICM schemes are compared to information outage probability for different system configurations. We consider the single-relay (Fig. 8), two-relay (Fig. 9), and three-relay (Fig. 10) half-duplex SSAF cooperative channels with different coding rates and constellation sizes. We use interleavers designed as in [19][18] with an additional constraint to transmit the systematic bits on the higher diversity blocks of the equivalent Matryoshka channels. We set the values of $\mathcal{E}_1 = 1$, and $\mathcal{E}_2 = \mathcal{E}_3 = 0.5$, so that the received average energy over all the time slots is invariant. We use rotations built using algebraic rotations from [21] (see Appendix I) that maximize the product distance over fading channels. In fact, as any space-time rotation with time spreading s can help providing diversity, these rotations are sufficient to achieve good performance in the absence of coding gain design criterion. The number of iterations between the detector and the decoder is fixed to 10. Fig. 8 shows the performance of ST-BICM over the single-relay SSAF channel using 64-QAM modulation and half-rate coding. Following $\delta_{max,1}$, no rotation is needed with the recursive systematic convolutional (RSC) code with generator polynomials $(23, 35)_8$, as the channel decoder with optimized interleaving is capable of recovering the maximum available diversity. For small to moderate signal-to-noise ratios, and due to noise amplification at the relay, precoding the signal constellation does not affect the performance. From moderate to high signal-to-noise ratios, a rotation yields a severe performance degradation (up to 5 dB). This is due to the fact that interference between symbols (due to the rotation) becomes too heavy for the decoder and thus affects the coding gain. This shows that, especially for high spectral efficiencies, spreading should be kept as small as possible so as to guarantee diversity, and it should even be avoided when not needed.

In Fig. 9, various coding strategies using RSC codes for the 2-relay SSAF protocol, all at an information rate of $R = 4/3$ b/s/Hz, are compared to Gaussian input outage probability. The first observation is that orthogonal coded schemes suffer from

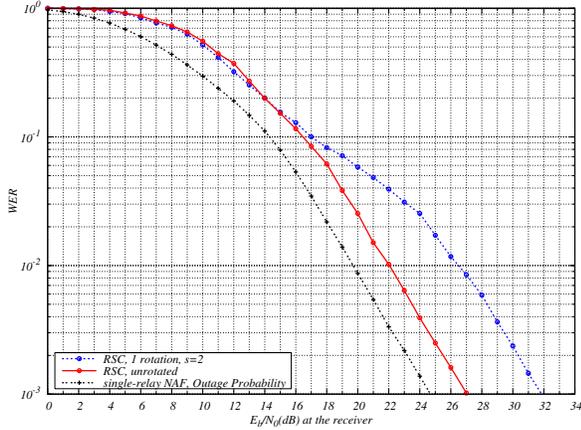


Fig. 8. Single-relay NAF cooperative channel, $R_c=1/2$ RSC $(23,35)_8$ code, 64-QAM modulation, $N = 1296$.

weak coding gains [7], although providing full diversity. For the curves employing the SSAF protocol, coding strategies following $\delta_{max,1}$ in (10), $\delta_{max,2}$ in (12), and the bound on the coding rate derived in Section IV-A.2. The best strategy is shown to be the $R_c = 2/3$ code with an $s = 2$ rotation with QPSK modulation in the three slots, following $\delta_{max,2}$. Note that the rate-1/3 code, that has a better free distance ($d_{free} = 7$), is outperformed by the precoding strategy with a weaker code (the rate-2/3 code has $d_{free} = 3$). Fig. 10 shows

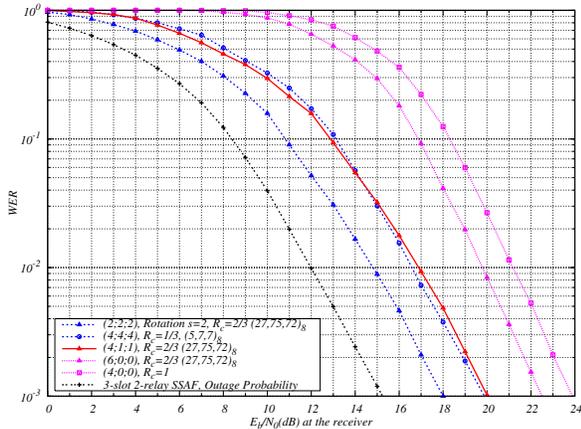


Fig. 9. 3-slot 2-relay SSAF protocol, $N = 1296$, $R = 4/3$ b/s/Hz. The set of spectral efficiencies over the cooperation frame is written as $(m_1; m_2; m_3)$, and QAM modulations are employed.

the performance of the SSAF with three relays using QPSK modulation and the half-rate $(133, 171)_8$ RSC code. The three strategies following $\delta_{max,2}$ and $\delta_{max,3}$ achieve full diversity with $R_c = 1/2$. Full precoding with $s = 4$, one-rotation and two-rotation precoding with $s = 2$ all achieve the same coding gain. This is probably because a powerful convolutional code is used. In case no precoder is available at the source and we want to transmit at the same coding rate, another option is to follow $\delta_{max,4}$ from (17), thus extending the cooperation frame with $\alpha = 2$ slots. This strategy allows to achieve full diversity without precoding, as shown with the dashed blue curve. Finally, it is important to note that simulations of coded

modulations with $(\beta + 1)^2 \times (\beta + 1)^2$ algebraic precoders as in [11] showed no gain with respect to $(\beta + 1) \times (\beta + 1)$ precoders in a presence of a powerful channel code, at a cost of a much higher detection complexity.

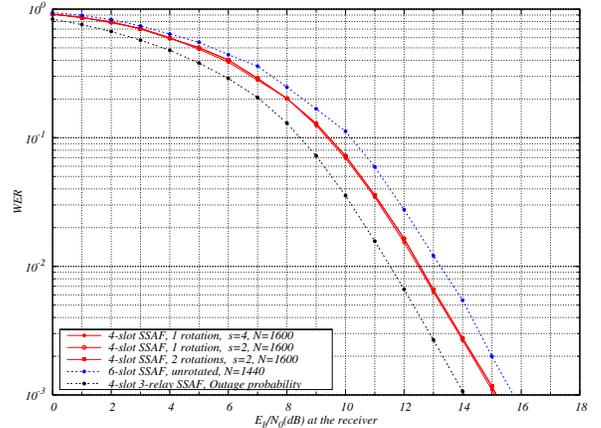


Fig. 10. 3-relay SSAF cooperative channel, QPSK modulation, $R_c = 1/2$ $(133, 171)_8$ RSC code.

VII. CONCLUSIONS

We studied coding strategies for the non-orthogonal amplify-and-forward half-duplex cooperative fading channel. We derive several bounds on diversity orders a coded modulation can achieve with low decoding complexity. We show that, given a coding rate, full diversity can be achieved either by space-time precoding, or by sending different spectral efficiencies over the slots, or even by stretching the cooperation frame (provided there are two relays or more). Moreover, although no closed-form expressions for the coding gain were derived, we showed that when using appropriate interleaving and space-time rotations, the diversity orders of the extrinsics at the output of the detector can be equal and thus performance is enhanced. Finally, performances close to outage probabilities for different number of relays, coding rates, and constellation sizes are shown.

APPENDIX I

EXAMPLES OF SPACE-TIME PRECODERS

The real 2×2 cyclotomic rotation from [21] can be written as:

$$\mathbf{S}_1 = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ \sin(\theta) & -\cos(\theta) \end{bmatrix} \quad (21)$$

with $\theta = 4.15881461$. Suppose now we have to transmit a half-rate code over the 3 relay SSAF channel. According to $\delta_{max,2}$, one rotation with $s = 2$ is sufficient. This gives the following space-time precoder:

$$\mathbf{S}_2 = \begin{bmatrix} \cos(\theta) & 0 & 0 & \sin(\theta) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \sin(\theta) & 0 & 0 & -\cos(\theta) \end{bmatrix} \quad (22)$$

According to $\delta_{max,3}$, we need two rotations with $s = 2$ each. This gives the following space-time precoder:

$$\mathbf{S}_3 = \begin{bmatrix} \cos(\theta) & 0 & 0 & \sin(\theta) \\ 0 & \cos(\theta) & \sin(\theta) & 0 \\ 0 & \sin(\theta) & -\cos(\theta) & 0 \\ \sin(\theta) & 0 & 0 & -\cos(\theta) \end{bmatrix} \quad (23)$$

The 4×4 rotation used in this paper is the Kruskemper rotation from [21] with normalized minimum product distance of 0.438993.

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