Precoder optimization for user fairness in shared relay channels with interference

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***Abstract*—This paper addresses the optimization of the linear precoder of a relay shared between several source/destination links interfering one with the others, in the aim of reaching fairness between users. The sources are not aware of the relay existence, and the destinations behave as in a relay channel with interference. Thus, the destinations do not try to decode other sources' messages and the relay is the only enabler for user cooperation. We consider several figures of merit for each user, such as the channel capacity or the residual interference level after MMSE filtering, which are combined by a user fairness utility function.**

I. Introduction

Linear precoding is the key technology for optimizing the channel capacity when channel state information is available at the transmitters side (CSIT). It becomes even more crucial when several nodes transmit concurrently, such as in multi-user MIMO [1] which is often addressed with interference alignment schemes [2], iterative optimization [3], or interference neutralization [4]. In the relaying context, linear precoder optimization has been derived for several scenarios with partial or full CSIT at the transmitting nodes, such as: the single-source single-destination relay channel capacity optimization through joint source-relay precoding [5][6], relay-only precoding [7], with multiple transmit antennas [8][9], or multiple relays shared for a single [10] or multiple [11] destinations, for a two-hop relay channel with one [12] or several [13] sources. Most of these papers consider extreme cases of the channel model in order to derive closed-form expressions of the precoder for the sum-rate capacity optimization.

This paper addresses the precoder optimization at a single relay shared between several source/destination pairs of a channel experiencing interference one on the other. One practical target is the deployment of relays at the cell-edge of a cellular system, where a large level of interference between neighboring base stations occurs. The aim of this paper is to provide a generic toolbox for the precoder optimization in the presented scenario. This paper does not address the optimization or computation of the capacity region of the interference channel involving cooperation between sources, or between sources and relay, or allowing a joint decoding of several sources signal at each destination. First, in section II, we present the channel model and notations. Then, the precoders optimized for several figure of merits are derived in section III. Finally, simulation results are shown in section IV.

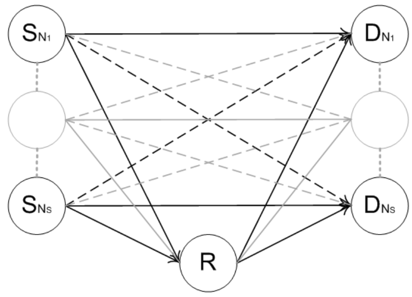

Fig. 1. Shared relay channel with interference and N_s pairs. Interference are illustrated with dashed lines.

II. Model of the shared relay channel with interference

We consider a shared relay channel with interference comprising N_s sources and N_s destinations, as illustrated in Fig.1. The sources are relay-unaware, i.e., they don't know the existence of the relay and don't adapt their transmission to the relay state, and have no channel state information (CSI).

The j-th destination has $N_{r,j}$ receive antennas and only tries to decode the j-th source message. We define $\mathbf{H}_j$ as the $N_{r,j} \times N_s$ channel observed between the sources and the j-th destination, and $\mathbf{A}_j$ as the $N_{r,j} \times N_s$ channel state information associated to $\mathbf{H}_j$ and available at the j-th destination. When full channel state information is assumed at the receivers (Full CSIR), a destination can estimate all the channels from all sources, i.e., it knows the structure of the pilot signals being for example orthogonal between the sources. When partial channel state information is assumed at the receiver (Partial CSIR), a destination can estimate the channel from its source only, and can build long term statistics of the channel from the other sources. This leads to

$$\begin{cases} \mathbf{A}_j &= \mathbf{H}_j, \quad \text{for full CSIR} \\ \mathbf{A}_j &= \mathbb{E}\left(\mathbf{H}_j\right)\bar{\mathbf{D}}_j + \mathbf{H}_j\mathbf{D}_j, \quad \text{for partial CSIR} \end{cases} \tag{1}$$

where $\mathbb{E}\left(\mathbf{H}_j\right)$ is the mathematical expectation of $\mathbf{H}_j$ which might be non-null in line-of-sight configurations, and $\mathbf{D}_j$ is a $N_s \times N_s$ selection diagonal matrix with one non-null element equal to 1 on the j-th diagonal position. By definition, we set $\bar{\mathbf{D}}_j = \mathbf{I} - \mathbf{D}_j$.

A relay is shared between the source/destination pairs in order to improve the useful signal and reduce the interference at each destination. The relay has N_t transmit antennas and applies a $N_t \times N_s$ precoder $\mathbf{P}$ on the N_s symbol estimations of the N_s sources symbols, or a subset of them. The relay obtains the CSI from all destinations in order to compute the linear precoder. As a remark, since we consider relay-unaware sources, the precoder cannot be performed jointly between the sources and the relay. Thus, the cooperation is only applied between the relay and the destinations, and the relay is the only enabler for cooperation between source/destination pairs.

We define $\mathbf{G}$ as the equivalent $N_s \times N_s$ channel observed between the sources and the relay, according to the selected relaying protocol, the symbol estimation at the relay, and the phase of the protocol. We define $\mathbf{F}_j$ as the equivalent $N_{r,j} \times N_t$ channel observed between the relay and the j-th destination, and is perfectly known at the j-th destination and at the relay.

The vector $\mathbf{Y}_j$ of length $N_{r,j}$ is observed by the j-th destination, and is defined by the channel model:

$$\mathbf{Y}_j = \mathbf{M}_j(\mathbf{P})\mathbf{z} + \boldsymbol{\eta}_j \tag{2}$$

where

$$\mathbf{M}_j(\mathbf{P}) = \mathbf{A}_j + \mathbf{F}_j\mathbf{P}\mathbf{G} \tag{3}$$

and where $\mathbf{z} = [z_1, \ldots, z_{N_s}]^T$, $z_j \in \Omega_j$, Ω_j being the signal alphabet of the j-th source. As a remark, the channel matrices $\mathbf{H}_j$, $\mathbf{F}_j$ and $\mathbf{G}$ are scaled in order to encompass both the fast fading and the path gain of the source-to-destination, relay-to-destination, or source-to-relay wireless links.

The covariance matrix of the noise $\boldsymbol{\eta}_j$ is equal to

$$\mathbb{E}\left[\boldsymbol{\eta}_j\boldsymbol{\eta}_j^\dagger\right] = \boldsymbol{\Sigma}_j \tag{4}$$

where $\dagger$ is the transpose conjugate operator. In this paper, the interference is by definition the part of the signal received by the j-th destination from the other sources and from the relay through the channel $\mathbf{M}_j\bar{\mathbf{D}}_j$. Furthermore, the noise encompasses the additive thermal noise with a $N_{r,j} \times N_{r,j}$ covariance matrix $2N_0\mathbf{I}$; and the residual of interference for which the destinations have no short-term statistics through the knowledge of $\mathbf{A}_i$, i.e., the residual interference signal equals $(\mathbf{H}_j - \mathbf{A}_j)\bar{\mathbf{D}}_j\mathbf{z}$. Thus, by using (1), the noise covariance matrix is defined by

$$\left\{\begin{array}{lcl} \boldsymbol{\Sigma}_j & = & 2N_0\mathbf{I}, \quad \text{for full CSIR} \\ \boldsymbol{\Sigma}_j & = & \mathbb{E}\left(\mathbf{H}_j\bar{\mathbf{D}}_j\mathbf{H}_j^\dagger\right) - \mathbb{E}\left(\mathbf{H}_j\right)\bar{\mathbf{D}}_j\mathbb{E}\left(\mathbf{H}_j\right)^\dagger \\ & + & 2N_0\mathbf{I}, \quad \text{for partial CSIR} \end{array}\right. \tag{5}$$

Finally, the destinations share their CSI matrices $\mathbf{M}_j(\mathbf{P})$ and $\boldsymbol{\Sigma}_j$ with the relay.

This channel model encompasses all relaying protocols in which the relay transmits the same symbols, or estimations of them, as a subset of sources, and on the same resource:

- For Decode-and-Forward (DF) relaying protocol involving the perfect knowledge of the symbols sent by a subset of sources, one set $\mathbf{G} = \boldsymbol{\Delta}_r$ which non-null elements are ones on the diagonal such as the non null elements of $\boldsymbol{\Delta}_r\mathbf{z}$ are the symbols known by the relay.
- For Amplify-and-Forward (AF) relaying protocols, the equivalent channel matrix $\mathbf{M}_j(\mathbf{P})$ classically encompasses the two-phase channel model and noise whitening. The channels matrices $\mathbf{H}_j$ and $\mathbf{F}_j$ include the combination of symbols made at the transmitters, such as space-time block codes.

In the following, we focus on the first case, but the results can be directly applied to the second one after a careful definition of the channel model.

III. Precoder optimization with user fairness

Let $f(\mathbf{P})$ be the figure of merit of the system performance, defined as

$$f(\mathbf{P}) = \left(\frac{1}{N_s}\sum_{j=1}^{N_s} f_j(\mathbf{P})^p\right)^{1/p} \tag{6}$$

where p is the parameter of the generalized mean, and $f_j(\mathbf{P})$ is the performance criterion for the j-th destination. One chooses $p = 1$ if the sum or the mean of the performance criterions has to be optimized. The minimum of the $f_j(\mathbf{P})$ values is obtained for $p \to -\infty$, while the maximum is obtained for $p \to +\infty$.

When considering an interference channel in a practical system with no joint decoding at the destinations, a fairness between users must be achieved, i.e., one usually does not want to drastically degrade the performance of one link for the benefit of another. Thus, we focus on the maximization of the minimal capacity among users.

The precoder $\mathbf{P}$ is optimized under a maximal transmit power constraint $h(\mathbf{P}) = \mathrm{Tr}\left(\mathbf{P}\mathbf{G}\mathbf{G}^\dagger\mathbf{P}^\dagger\right) - 1 \leq 0$. This leads to the following Lagrange multipliers system

$$\frac{\partial f(\mathbf{P})}{\partial \mathbf{P}^*} = \lambda\frac{\partial h(\mathbf{P})}{\partial \mathbf{P}^*} \quad \text{and} \quad \frac{\partial h(\mathbf{P})}{\partial \mathbf{P}^*} \leq 0 \tag{7}$$

where λ is the Lagrange multiplier and from (6), we have

$$\frac{\partial f(\mathbf{P})}{\partial \mathbf{P}^*} = \frac{f(\mathbf{P})^{1-p}}{N_s}\sum_{j=1}^{N_s} f_j(\mathbf{P})^{p-1}\frac{\partial f_j(\mathbf{P})}{\partial \mathbf{P}^*}. \tag{8}$$

The system (7) usually does not have a closed form solution, and the precoder $\mathbf{P}$ can be obtained by an iterative projected Gradient descent by applying the algorithm

$$\mathbf{P} \leftarrow \mathbf{P} + \frac{\mu}{\min\left(\mathrm{Tr}\left(\mathbf{P}\mathbf{G}\mathbf{G}^\dagger\mathbf{P}^\dagger\right), 1\right)} \times \frac{\partial f(\mathbf{P})}{\partial \mathbf{P}^*} \tag{9}$$

where μ is a typical parameter for the Gradient descent, and where the solution is projected in the space of solutions satisfying the constraint $h(\mathbf{P}) \leq 0$.

In the following, we derive the expression of $\partial f_j(\mathbf{P})/\partial \mathbf{P}^*$ for three widely used criterions: the channel capacity, the discrete input mutual information under a Gaussian approximation of the interference, and the Signal to Interference plus Noise Ratio (SINR) after Minimum Mean Square Error (MMSE) filtering.

A. Derivative of the criterion

In order to obtain the derivative $\partial f_j(\mathbf{P})/\partial \mathbf{P}^*$ according to the complex-valued matrix $\mathbf{P}$, we apply the methodology described in [14], i.e., by using the following property

$$df_j(\mathbf{P}) = \mathrm{Tr}\left(\boldsymbol{\Gamma}_0^T d\mathbf{P} + d\mathbf{P}^\dagger\boldsymbol{\Gamma}_1\right) \Rightarrow \frac{\partial f_j(\mathbf{P})}{\partial \mathbf{P}^*} = \boldsymbol{\Gamma}_1 \tag{10}$$

where the differential $df_j(\mathbf{P})$ is obtained from the toolbox of differential calculus. In this paper, we use the following non-trivial expressions

$$\begin{aligned} d\left(\mathrm{Tr}(\mathbf{Z})\right) &= \mathrm{Tr}(d\left(\mathbf{Z}\right)) && (11)\\ d(\mathbf{Z}^{-1}) &= -\mathbf{Z}^{-1}d(\mathbf{Z})\mathbf{Z}^{-1} && (12)\\ d(\log|\mathbf{Z}|) &= \mathrm{Tr}(\mathbf{Z}^{-1}d\left(\mathbf{Z}\right)) && (13)\end{aligned}$$

where $|\mathbf{Z}|$ is the determinant of the matrix $\mathbf{Z}$.

B. Optimization of the Channel Capacity

In this section we define $f_j(\mathbf{P}) = C_j(\mathbf{P})$ as the channel capacity observed at the j-th destination, where the interference is considered as a spatially correlated noise. The useful signal covariance matrix for the j-th link is $\mathbf{M}_j(\mathbf{P})\mathbf{D}_j\mathbf{M}_j(\mathbf{P})^\dagger$ while the interference plus noise signal is spatially correlated, with covariance matrix $\mathbf{\Lambda}_j\mathbf{\Lambda}_j^\dagger$:

$$\mathbf{\Lambda}_j\mathbf{\Lambda}_j^\dagger = \mathbf{M}_j(\mathbf{P})\bar{\mathbf{D}}_j\mathbf{M}_j(\mathbf{P})^\dagger + \mathbf{\Sigma}_j \quad (14)$$

Thus, by using $\mathbf{\Lambda}_j^{-1}$ as a whitening filter, one obtain the expression of the capacity of the virtual MIMO channel with interference [15]

$$\begin{aligned} C_j(\mathbf{P}) &= \log_2\left|\mathbf{I} + \mathbf{\Lambda}_j^{-1}\mathbf{M}_j(\mathbf{P})\mathbf{D}_j\mathbf{M}_j(\mathbf{P})^\dagger\mathbf{\Lambda}_j^{-\dagger}\right| && (15)\\ &= \log_2\left|\mathbf{M}_j(\mathbf{P})\mathbf{M}_j(\mathbf{P})^\dagger + \mathbf{\Sigma}_j\right| \\ &\quad - \log_2\left|\mathbf{M}_j(\mathbf{P})\bar{\mathbf{D}}_j\mathbf{M}_j(\mathbf{P})^\dagger + \mathbf{\Sigma}_j\right| && (16)\end{aligned}$$

By using (10), the derivation leads to

$$\begin{aligned}\frac{\partial C_j(\mathbf{P})}{\partial \mathbf{P}^*} &= \frac{1}{\ln(2)}\mathbf{F}_j^\dagger\Big[\left(\mathbf{M}_j(\mathbf{P})\mathbf{M}_j(\mathbf{P})^\dagger + \mathbf{\Sigma}_j\right)^{-1}\mathbf{M}_j(\mathbf{P}) \\ &\quad - \left(\mathbf{M}_j(\mathbf{P})\bar{\mathbf{D}}_j\mathbf{M}_j(\mathbf{P})^\dagger + \mathbf{\Sigma}_j\right)^{-1}\mathbf{M}_j(\mathbf{P})\bar{\mathbf{D}}_j\Big]\mathbf{G}^\dagger \quad (17)\end{aligned}$$

The precoder $\mathbf{P}$ is optimized at the relay by using (9), (8) and (17) with $f_j(\mathbf{P}) = C_j(\mathbf{P})$. The minimum of the capacities is maximized by choosing $p \to -\infty$, which guarantees fairness between users.

Optimal or sub-optimal closed form expressions can be obtained in some particular cases. For example, when the power received from the sources is negligible with respect to the power received from the relay, the precoder optimization is similar to the one obtained for a downlink multi-users system, or equivalently for a system with no direct source to destination link [13].

For the other cases, which are the targets of this paper, where a destination receives non-negligible power from the sources and the relay, the iterative estimation of the optimal $\mathbf{P}$ is used. It can be shown that (15) is concave for finite p values, which ensures convergence. Thus, in order to improve the convergence of the iterative descent, we consider low negatives values for p (e.g., $p = -10$), which approximates the $min(.)$ function by a smoother one.

C. Optimization of the Discrete input Mutual information with Gaussian approximation of the interference

In this section, we consider the mutual information $f_j(\mathbf{P}) = \mu_j(\mathbf{P})$ constrained to a finite alphabet input for the j-th source, and making a Gaussian approximation of the interference generated by other sources symbols. This corresponds to the case where a destination or the relay has no knowledge of the alphabet used by the other sources. The equivalent channel model is

$$\mathbf{Y}_j = \mathbf{M}_j(\mathbf{P})\mathbf{D}_j\mathbf{z} + \boldsymbol{\eta}_j' \quad (18)$$

where $\boldsymbol{\eta}_j'$ is the Gaussian approximation of the interference plus noise signal

$$\boldsymbol{\eta}_j' \simeq \mathbf{M}_j(\mathbf{P})\bar{\mathbf{D}}_j\mathbf{z} + \boldsymbol{\eta}_j \quad (19)$$

and

$$\mathbb{E}\left[\boldsymbol{\eta}_j'\boldsymbol{\eta}_j'^\dagger\right] = \mathbf{\Lambda}_j\mathbf{\Lambda}_j^\dagger \quad (20)$$

The mutual information $\mu_j(\mathbf{P})$ is expressed as

$$\mu_j(\mathbf{P}) = \mathbb{E}_{\mathbf{z}_j,\boldsymbol{\eta}_j}\left[\log_2\left(\frac{p(\mathbf{Y}_j|\mathbf{z}_j)}{\mathbb{E}_{\mathbf{z}_j''}\left[p(\mathbf{Y}_j|\mathbf{z}_j'')\right]}\right)\right] \quad (21)$$

where

$$p(\mathbf{Y}_j|\mathbf{z}_j') \propto e^{-\|\mathbf{\Lambda}_j^{-1}(\mathbf{M}_j(\mathbf{P})(\mathbf{z}_j-\mathbf{z}_j')+\boldsymbol{\eta}_j)\|^2} \quad (22)$$

The derivation of $\mu_j(\mathbf{P})$ leads to

$$\frac{\partial\mu_j(\mathbf{P})}{\partial\mathbf{P}^*} = \mathbb{E}_{\boldsymbol{\eta}_j,\mathbf{z}_j,\mathbf{z}_j'}\left[\frac{\frac{\partial\|\mathbf{\Lambda}_j^{-1}(\mathbf{M}_j(\mathbf{P})(\mathbf{z}_j-\mathbf{z}_j')+\boldsymbol{\eta}_j)\|^2}{\partial\mathbf{P}^*}p(\mathbf{Y}_j|\mathbf{z}_j')}{\ln(2)\mathbb{E}_{\mathbf{z}_j''}\left[p(\mathbf{Y}_j|\mathbf{z}_j'')\right]}\right] \quad (23)$$

where

$$\begin{aligned}&\frac{\partial\|\mathbf{\Lambda}_j^{-1}(\mathbf{P})(\mathbf{M}_j(\mathbf{P})(\mathbf{z}_j-\mathbf{z}_j')+\boldsymbol{\eta}_j)\|^2}{\partial\mathbf{P}^*} \\ &= \mathbf{F}_j^\dagger(\mathbf{\Lambda}_j(\mathbf{P})\mathbf{\Lambda}_j(\mathbf{P})^\dagger)^{-1}(\mathbf{M}_j(\mathbf{P})(\mathbf{z}_j-\mathbf{z}_j')+\boldsymbol{\eta}_j) \\ &\quad\times\big((\mathbf{z}_j-\mathbf{z}_j')^\dagger - (\mathbf{M}_j(\mathbf{P})(\mathbf{z}_j-\mathbf{z}_j')+\boldsymbol{\eta}_j)^\dagger \\ &\quad\times(\mathbf{\Lambda}_j(\mathbf{P})\mathbf{\Lambda}_j(\mathbf{P})^\dagger)^{-1}\mathbf{M}_j(\mathbf{P})\bar{\mathbf{D}}_j\big)\mathbf{G}^\dagger \quad (24)\end{aligned}$$

The precoder $\mathbf{P}$ is optimized at the relay by using (8), (9) and (24) with $f_j(\mathbf{P}) = \mu_j(\mathbf{P})$. The optimization of $\mu_j(\mathbf{P})$ at the relay has the advantage to accurately model the behavior of modern coding schemes including rate matching and hybrid ARQ, and the drawback of a high complexity.

D. Optimization of the SINR under MMSE filtering

In this section, we assume that a MMSE filter is applied at each destination according to its level of CSI. The precoder $\mathbf{P}$ must be chosen so as to maximize the SINR or equivalently minimize the level of remaining interference $f_j(\mathbf{P}) = \epsilon_j(\mathbf{P})$ at the output of the MMSE filter. The MMSE filter at the j-th destination is expressed as

$$\mathbf{W}_j(\mathbf{P}) = \mathbf{M}_j(\mathbf{P})^\dagger\left[\mathbf{M}_j(\mathbf{P})\mathbf{M}_j(\mathbf{P})^\dagger + \mathbf{\Sigma}_j\right]^{-1} \quad (25)$$

The remaining interference level is

$$\epsilon_j(\mathbf{P}) = \mathrm{Tr}\left[\mathbf{D}_j - \mathbf{D}_j\mathbf{W}_j(\mathbf{P})\mathbf{M}_j(\mathbf{P})\mathbf{D}_j\right] \quad (26)$$

The derivation of $\epsilon_j(\mathbf{P})$ leads to

$$\frac{\partial\epsilon_j(\mathbf{P})}{\partial\mathbf{P}^*} = \mathbf{F}_j^\dagger\mathbf{W}_j^\dagger(\mathbf{P})\mathbf{D}_j\left[\mathbf{W}_j(\mathbf{P})\mathbf{M}_j(\mathbf{P}) - \mathbf{I}\right]\mathbf{G}^\dagger \quad (27)$$

The precoder $\mathbf{P}$ is optimized at the relay by using (8), (9) and (27) with $f_j(\mathbf{P}) = \epsilon_j(\mathbf{P})$.

IV. Simulation Results

We consider an shared relay channel with interference comprising $N_s = 2$ source/destination pairs, each having 1 transmit and $N_{r,j} = 1$ receive antenna. We consider a phase of a DF protocol, during which the relay's transmits the same symbols as the subset of sources defined by the relaying configuration indicator matrix $\mathbf{\Delta}_r$. When $\mathbf{\Delta}_r = \mathrm{diag}([1\ 0])$, only the symbols of the first source are relayed; when $\mathbf{\Delta}_r = \mathrm{diag}([0\ 1])$, only the symbols of the second source are relayed; and when $\mathbf{\Delta}_r = \mathrm{diag}([1\ 1])$, the symbols of both sources are relayed by the relay. The symbols are precoded by $\mathbf{P}$ before transmission on the N_t transmit antennas of the relay.

The long-term signal to interference plus noise ratio (SINR) is defined without taking the relay effect into account. The signal to noise ratio (SNR) is the ratio between the received useful signal power and the thermal noise power and is set to $SNR = 30dB$ at each destination. The interference channel is symmetric, i.e., each destination experiences the same level of interference from the other sources. We do not consider any spatial correlation for the equivalent MIMO channel.

In Fig. 2, we consider the QPSK input mutual information (21) obtained at destination 1 with full CSI knowledge at the relay and destinations. At the j-th destination, the average power P_s received from the j-th source and from the relay P_r are equal. The performance of the interference channel without relay is plotted as a function of the SINR. For low SINR levels, the system is interference limited and high gains are expected from interference neutralization, while for SINR levels approaching the SNR, the potential gain obtained from interference neutralization vanishes.

When no optimized precoder is used at the relay and when $\mathbf{\Delta}_r = \mathrm{diag}([1\ 0])$, the useful signal is boosted and the performance improved. When $\mathbf{\Delta}_r = \mathrm{diag}([0\ 1])$, the interference signal is boosted and the performance highly degraded. From that observation, we understand that the non-precoded use of a relay improves a targeted user while degrading the others. When $\mathbf{\Delta}_r = \mathrm{diag}([1\ 1])$, both signals are boosted, which asymptotically leads to a 3dB boosting gain for very low SINR values where the interference injected by the relay is negligible with respect to the interference from the other sources. When the SINR increases up to the SNR, the performance saturates since the relay generates as much useful signal as interference.

We then consider the use of an optimized precoder according to (9), (8) and (17), i.e., maximizing the minimal capacity instead of the mutual information. The mismatch between a capacity optimizing precoder and a mutual information optimizing precoder is negligible and not illustrated here for sake of clarity. The capacity-optimizing precoder computation is far less complex and more practical for an implementation in a relay. A gain is obtained for any relaying configuration. The performance for $\mathbf{\Delta}_r = \mathrm{diag}([1\ 0])$ and $\mathbf{\Delta}_r = \mathrm{diag}([0\ 1])$ are equivalent, which means that, with the assumed symmetric interference channel configuration, the precoder optimization allows to equalize the benefit taken from power boosting and from interference neutralization. When the relay transmits signal of both sources, each destination takes benefit from both power boosting and interference neutralization, which further improves the performance. We observe that the channel capacity gain is around 180% for a $SINR = 0dB$ when the relay transmits signal from both sources. In that case, the relay helps to almost remove all the interference from one link to the other. Equivalent behaviors would be observed for the Partial CSI case.

In Fig. 3, we show the iso-capacity curves for a capacity of 2 b/s/Hz for the destination 1 as a function of the SINR and the P_r/P_s ratio. We consider several relaying and antenna configurations at the relay. First, for a relay equipped with 1 antenna, we observe the same behavior for the configurations $\mathbf{\Delta}_r = \mathrm{diag}([1\ 0])$ and $\mathbf{\Delta}_r = \mathrm{diag}([0\ 1])$, which confirms that, under the user fairness assumption, the interference neutralization is roughly as efficient as the power boosting. A substantial gain is observed for the $\mathbf{\Delta}_r = \mathrm{diag}([1\ 1])$ configuration taking benefit of both boosting and interference neutralization. The performance is improved by increasing the number of antennas at the relay. The relay transmit power being normalized from one configuration to the other, the gain comes from two aspects: diversity improvement and channel orthogonalization from interference neutralization.

In order to illustrate the diversity improvement behavior, we plot in Fig.4 the outage probability for a variable number of pairs and antennas at the relay for a relaying configuration $\mathbf{\Delta}_r = \mathrm{diag}([1 \cdots 1])$, i.e., when the relay transmits all symbols from all sources. We observe that the diversity order of the outage probability is $N_t + 1$, which means that the relay precoder provides full transmit diversity to all links. The theoretical justification of this result is still an open problem and left for future work. By increasing the number of sources, we observe performance degradation, which is due to the relay power sharing and remaining level of interference. This degradation is limited when the number of antennas of the relay increases, which is explained by an improved orthogonality of the equivalent channel by the relay precoder.

Additional results would show how the precoder can help in improving the non-coded system performance by using the MMSE-based precoder optimization. In practice, this precoder optimization can be used for very high data rates while the capacity-optimizing precoder is preferable as soon as an error-correcting code is used.

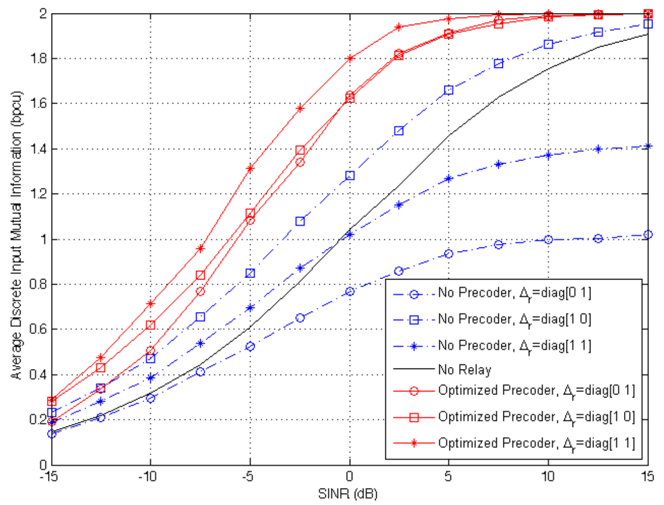

Fig. 2. Mutual information with QPSK input for a 2-pairs IRC channel with full CSI at the destinations, with and without optimized precoding at the relay. The destinations have 2 receive antennas, and the relay 2 transmit antennas. The interference channel is symmetric and $SNR = 30dB$ and $P_r = P_s$.

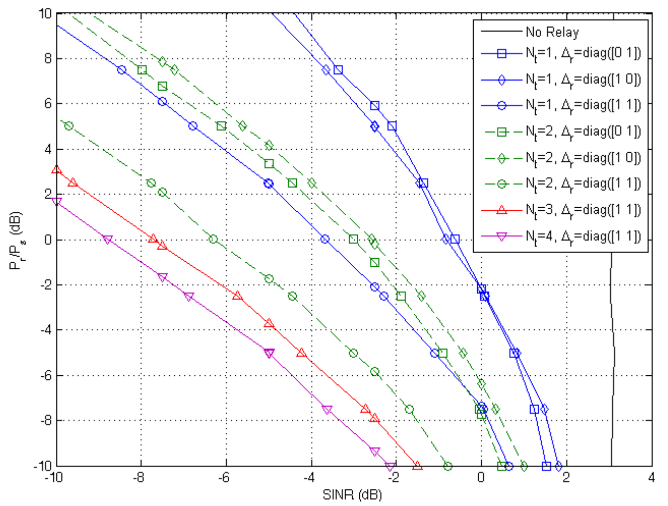

Fig. 3. Iso-Capacity curves for a capacity of 2b/s/Hz as a function of the SINR and P_r/P_s, with different antenna and relaying configuration at the relay and full CSI knowledge at the destinations. $SNR = 30dB$.

V. Conclusions

In this paper, we have presented a precoder optimization framework for a relay shared between several source/destination links interfering one with the other, in the aim of reaching fairness between users. As no closed-form expression of the precoder exists for the general case, the precoder is evaluated by a projected gradient descent under a maximal transmit power constraint at the relay. Simulations results show that transmit diversity improvement and interference neutralization are achieved at the same time. Future work will include the theoretical analysis of the diversity behaviors for this system, as well as a precoder optimization for the capacity region of the IRC channel.

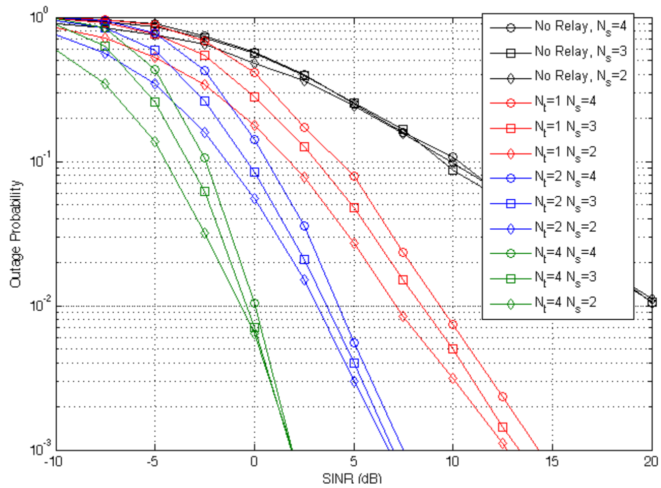

Fig. 4. Outage Probability for a data rate of 2b/s/Hz, for different number of sources N_s and number of transmit antennas at the relay N_t

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