Consensus- based decentralization of interior point methods for heterogeneous networks energy saving

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Abstract—In this paper, we propose a new decentralized consensus-based energy saving technique that improves the energy efficiency of a cellular network, with a focus on femto base station networks. The energy saving is obtained by a log barrier optimization on the total transmit power, with a long-term or short-term coverage constraint for each cell. The common coordination parameter of the log-barrier method is approximated at each node by the result of a consensus algorithm, which allows for tracking the topology variations in a fully decentralized fashion. Simulation shows that the proposed technique doubles the energy saving efficiency of the centralized energy saving.

I. Introduction

In future wireless communication systems, energy saving is an important target. In cellular systems, the downlink energy saving consists of adjusting the power of base stations so as to meet some coverage/capacity constraints for its users and minimize the overall radiated power in the network. This minimization is performed either by turning off some base stations of the network (hard energy saving) or by gradually reducing the radiated power of the base stations in order to meet users coverage constraints (soft energy saving). The advantage of the latter energy saving technique being the minimization of coverage holes that may be introduced by the hard energy saving, and more robustness to topology change and users mobility.

In this paper, we are focusing on soft energy saving for heterogeneous networks including several types of nodes such as macro, pico or femto base stations and relays. The coordination of an heterogeneous deployment is difficult because of its random topology and specific architecture constraints. Thus, it is important to develop stable and decentralized algorithms achieving good energy efficiency when compared to their centralized counterparts. The proposed decentralized power saving technique is obtained by combining the log-barrier interior point optimization technique [5] and average consensus techniques [2]. The decentralized algorithm can achieve both energy saving and fast convergence/adaptation to the wireless topology change.

The outline of the paper is as follows: In section II, the energy saving optimization problem is stated and the centralized log barrier optimization technique is presented. In section III, consensus-based log barrier techniques are formulated to handle energy saving in the heterogeneous network with dynamic topology. These energy saving techniques are evaluated through system level simulations in section IV. Finally, conclusions of the paper are drawn in section V.

II. Centralized interior point based energy saving

Consider a network of N nodes, each node is transmitting with a power P_i where $i = 1, \cdots, N$. The downlink energy saving optimization problem is formulated as the minimization of the sum power of the network, under the constraint that each user equipment (UE) j of the i-th node has a signal to interference plus noise ratio (SINR) above a minimum required SINR γ_i. The optimization problem is set up in equation (1).

$$\begin{cases} \text{minimize } \sum_{i=1}^{N} P_i \\ \text{subject to } \forall 0 < i \leq N, \dfrac{\alpha_{i,u,i}P_i}{\sum_{j \in \Omega(i)} \alpha_{i,u,j}P_j + N_0} \geq \gamma_i \end{cases} \quad (1)$$

where $\Omega(i)$ is the index set of the i-th node radio neighborhood, and N_0 is the noise variance. The path gain $\alpha_{i,u,j}$ is the path gain between the j-th node and the cell edge UE u of the i-th node, and is obtained from downlink measurements at the i-th node.

For a short-term coverage control, the cell edge user u is defined as the user with the worst measured SINR in the cell i at a given time. For a long-term coverage control, the path gains associated to the worst last position of a UE in the cell is stored and used for the optimization.

The energy saving optimization problem is linear in the transmit powers P_i and in the coverage constraints. Thus, the centralized log barrier interior point optimization can be used [5].

A. Centralized log barrier interior point method

The basic principle of log barrier interior point method is to solve the linear optimization problem of equation (1) by including the constraints of the optimization into the following augmented objective function :

$$\text{minimize } J(t) = \sum_{i=1}^{N} P_i + \frac{1}{t} \sum_{i=1}^{N} \log\left(\lambda_i(P_1, \cdots, P_N)\right) \quad (2)$$

where t is the common coordination parameter of the log-barrier method. The functions $\lambda_i(P_1, \cdots, P_N)$, denoted λ_i in the rest of the paper, are the inverse of the constraints of the i-th node:

$$\lambda_i = \left[P_i - \frac{\gamma_i}{\alpha_{i,u,i}}\left(\sum_{j \in \Omega(i)} \alpha_{i,u,j}P_j + N_0\right)\right]^{-1} \quad (3)$$

The log barrier interior point method solves a sequence of minimizations of $J(t)$ with increasing values of the parameter

t. The algorithm is centralized because information on the current value of t is maintained in a central node of the network. Starting with a low value of t allows to give a large weight to the constraint at the beginning of the optimization process, while decreasing it through time allows to converge to the solution of the optimization that satisfy the constraints. Thus, the log barrier procedure starts with a low value of the parameter t, increases t step after step, and stops when the value of t is above $\frac{N}{\epsilon}$, where ϵ is a precision threshold.

The derivation of $J(t)$ in (2) with respect to P_ℓ gives the following updated $\lambda_\ell(k+1)$ at the $k+1$-th iteration:

$$\lambda_\ell(k+1) = t(k) + \sum_{j \in \Omega(\ell)} \frac{\gamma_j \alpha_{j,u,\ell}}{\alpha_{j,u,j}} \lambda_j(k) \qquad (4)$$

$$= t(k) + \sum_{j \in \Omega(\ell)} \alpha_{j,u,\ell} \hat{\lambda}_j(k) \qquad (5)$$

where

$$\hat{\lambda}_j(k) = \frac{\gamma_j \lambda_j(k)}{\alpha_{j,u,j}}. \qquad (6)$$

The radiated power of the ℓ-th node is set up to the centralized log barrier power level $P_\ell(k+1)$, which is obtained from the parameter $\lambda_\ell(k+1)$ as

$$P_\ell(k+1) = \frac{\gamma_\ell}{\alpha_{\ell,u,\ell}} \left(\sum_{j \in \Omega(\ell)} \alpha_{\ell,u,j} P_j(k) + N_0 \right) + \frac{1}{\lambda_\ell(k+1)} \qquad (7)$$

The update rule (4) is the sum of the common parameter $t(k+1)$ that is driven by the central node of the network and the neighbor correction terms $\hat{\lambda}_j(k)$ weighted by the path gain $\alpha_{j,u,\ell}$. Thus, the update transmit power of the ℓ-th node in (7) can be computed from downlink measurements $\alpha_{\ell,u,j}$ obtained at the ℓ-th node, and from the neighboring nodes transmit power $P_j(k)$. The most decentralized implementation of the log-barrier optimization implies a broadcast of the updated values $\lambda_i(k+1)$ and $P_i(k+1)$ by each node and a distribution of the measurements $\alpha_{i,u,j}$ from node i to node j. For each iteration $k+1$ of the energy saving procedure, each node ℓ updates its power and the parameter $\lambda_\ell(k+1)$ by:

- Obtaining the measurements $\alpha_{\ell,u,j}$ from its u-th UE
- Obtaining the measurements $\alpha_{j,u,\ell}$ from each neighboring nodes j
- Obtaining parameters $P_j(k)$, $\hat{\lambda}_j(k)$ and $\alpha_{\ell,u,j}$ from each neighboring nodes j and parameter $t(k)$ from the central node of the network.
- Computing $\lambda_\ell(k+1)$ and $P_\ell(k+1)$ as given in (4) and (7).
- Broadcasting $\lambda_\ell(k+1)$ and $P_\ell(k+1)$ to its neighboring nodes

The central node of the network waits for each node in the network to update its power, then the common parameter t is increased and a new iteration of the log barrier method is run again. In the following, we apply a consensus algorithm for the common cooperation parameter t, in order to implement the log barrier optimization in a fully decentralized fashion.

III. De-centralized log-barrier optimization with consensus techniques

A. De-centralization for a fixed topology

First, a decentralization of the log barrier energy saving procedure is proposed for network with a fixed topology. As seen in the previous section, the main drawback of the centralized log barrier technique is that the central node of the network needs to provide the same common parameter t to all the nodes of the network and waits for the nodes of the network to update their power before increasing the common parameter t. In order to develop decentralized energy saving procedure that overcome this drawback, it is proposed to update a node specific parameter in the iteration k, i.e. $t_\ell(k)$ for the ℓ-th node, along with the log barrier iterations given in (4) and (7).

This update is done through *consensus iteration* procedure and is involving the following basic steps:

- Exchanging the parameter $t_\ell(k)$ with its radio neighbors.
- Updating the parameter $t_\ell(k)$ using the received parameters $t_j(k-1)$ from the neighbors with consensus update function.
- Performing the log barrier power saving iteration based on the updated parameter $t_\ell(k)$.

The general form of the consensus update function is shown in the equation below

$$t_\ell(k+1) = f\left(t_{\Omega_\ell(1)}(k), \cdots, t_{\Omega_\ell(|\Omega_i|)}(k), t_\ell(k)\right) \qquad (8)$$

where $|\Omega_\ell|$ is the number of the radio neighbors of the node ℓ. The function $f(.)$ is the consensus function that determines the consensus update rule of the log barrier parameter t. The simplest consensus function $f(.)$ is related to the topology of the network by the following relationship [1]

$$t_\ell(k+1) = \sum_{j \in \Omega(\ell)} w_{j,\ell} t_j(k) \qquad (9)$$

The parameters $w_{j,\ell}$ are the elements of the connectivity graph Laplacian, i.e. $w_{j,\ell} = \frac{1}{|\Omega_\ell|}$. For connected graphs, the consensus iteration given in (9) converges to the average of the parameters t_ℓ [1]. Unfortunately, applying this consensus iteration implementation is not robust to the wireless network topology changes. In the following, we adapt the consensus approach to the log-barrier behavior in order to obtain a decentralized algorithm robust to topology changes.

B. A random-topology robust decentralization of the log-barrier method

When new nodes are turned on or off in the network, the interference map changes drastically but locally. Each change implies the re-computation of the whole parameters when implementing the centralized version of the log barrier algorithm, i.e., the central entity has to reset the common coordination parameter t to a low value. A local topology change only impacts the interference perceived by the neighboring nodes, and should not imply a transmit power update of the more distant nodes.

In heterogeneous networks, these dynamic topology changes can be frequent and decrease the achieved energy saving efficiency of the centralized log barrier method. Furthermore, the architecture of the network does not usually support a centralized optimization. It was shown in section (III-A) that consensus iterations over the common parameter t can solve the problem. However, this consensus update needs to be adapted to the log barrier iterations. Indeed, having high values of parameters t_ℓ in a region of the wireless network implies a high stability in the optimization process. If one node turns on or off, its impact on the interference is high, and the optimization should be recomputed, but only for neighboring nodes. In the sections below, four consensus modes are proposed to achieve the goal of tracking dynamic topology change in the network while performing log barrier iterations.

1) Consensus mode 1: In order to relax the optimization between nodes when the state of one of them have changed, we apply the consensus iterations on the inverses of the log barrier parameter t_ℓ. The corresponding consensus update rule is given as

$$\frac{1}{t_\ell(k+1)} = \frac{1}{\mid \Omega_i \mid +1} \left(\sum_{j \in \Omega(i)} \frac{1}{t_j(k)} + \frac{1}{t_\ell(k)} \right) \quad (10)$$

In this case, each node propagates its log barrier parameter to the neighboring nodes. The base ℓ calculates its new parameter $t_\ell(k)$ as the inverse of the average of the inverses of the received parameters $t_j(k)$. When the state of one node j changes, it sets its barrier parameter $t_j(k)$ to a low value that will propagate to the neighboring nodes via the consensus iteration.

2) Consensus mode 2: Radio neighbors do not usually generate the same level of interference, and the re-computation of the optimization must be prioritized by highest interferers. The consensus update rule can be explicitly tightened to the received powers of the neighboring nodes:

$$\frac{1}{t_\ell(k+1)} = \frac{1}{\sum_{j \in \Omega(\ell)} P_j \alpha_{\ell,u,j} + P_i \alpha_{\ell,u,\ell}} \left(\sum_{j \in \Omega(\ell)} \frac{P_j \alpha_{\ell,u,j}}{t_j(k)} + \frac{P_\ell \alpha_{\ell,u,\ell}}{t_\ell(k)} \right) \quad (11)$$

The consensus mode 2 allows more dynamical tracking of the variations of the topology of the network but may lead to low convergence speed compared to the consensus mode 1.

3) Consensus mode 3: For consensus mode3, the consensus iteration of equation(11) is penalized with a positive parameter $\mu \leq 1$ in order to improve its convergence

$$\frac{1}{t_\ell(k+1)} = \frac{\mu}{\sum_{j \in \Omega(\ell)} P_j \alpha_{\ell,u,j} + P_\ell \alpha_{\ell,u,\ell}} \left(\sum_{j \in \Omega(\ell)} \frac{P_j \alpha_{\ell,u,j}}{t_j(k)} + \frac{P_\ell \alpha_{\ell,u,\ell}}{t_\ell(k)} \right) \quad (12)$$

4) Consensus mode 4: In consensus mode 4, the weighting of the average consensus is normalized with respect to the maximum received power from the neighborhood of the node ℓ. The consensus update rule is given as:

$$\frac{1}{t_\ell(k+1)} = \frac{\mu}{(\mid \Omega_\ell \mid +1) \max_{j \in \Omega(\ell)}(P_j \alpha_{\ell,u,j}, P_i \alpha_{\ell,u,\ell})} \left(\sum_{j \in \Omega(\ell)} \frac{P_j \alpha_{\ell,u,j}}{t_j(k)} + \frac{P_i \alpha_{\ell,u,\ell}}{t_\ell(k)} \right) \quad (13)$$

This consensus update rule allows the tracking of topology variations conditioned on the the maximum interference level perceived from the neighborhood of the ℓ-th node. If the level of the interference is high, the convergence speed is low and if the interference level is low the convergence speed is high. In section (IV), simulation scenarios are described for the validation of the proposed decentralized, consensus based, log barrier energy saving technique for a typical indoor femtocell network. The four consensus modes are compared in terms of overall convergence speed for different deployment topologies and energy saving efficiency with respect to the centralized log barrier energy saving.

IV. SIMULATION RESULTS

In this section, simulation scenario is setup to compare the different energy saving techniques proposed previously. First, baseline performance results of the centralized log barrier energy saving technique are presented for the fixed topology scenario. In the fixed topology scenario, 25 femto base stations are randomly deployed in a square area of 50×50 square meters. Each femto base station is placed at random in an apartment of size 10×10 square meters. The indoor path loss is modeled through standard Motley Keenan propagation model [6] that adds deployment dependent indoor walls attenuation to the free space propagation. We have considered three typical values for walls attenuations in the simulations, i.e. $w = 0, 10, 20$ dB. Each femto base station ℓ maintains the list of the neighboring femto base stations Ω_ℓ that are received with SINR level above -10dB in its coverage region. The radio parameters of the femto base station and the indoor channel are summarized in Table (I).

Parameters	Assumptions
Indoor path loss model	$L = 38 + 30 log(r)$, r in meters
Shadowing	no shadowing
Indoor walls attenuation	$0, 10, 20$ dB
Carrier frequency	2GHz
maximum transmit power	20dBm
Shadowing	No shadowing
Antenna gain	5dBi
UEs noise factor	9 dB

TABLE I
SIMULATION ASSUMPTIONS

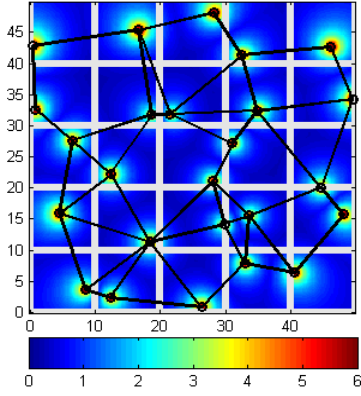Two system level simulation scenarios are considered in this paper:

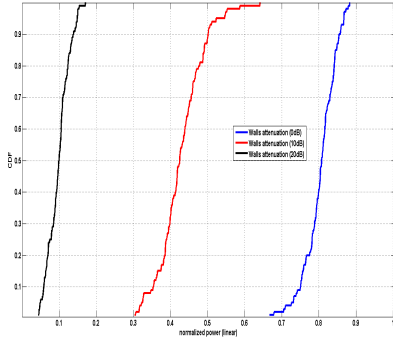Fig. 1.   Typical coverage map for the femto scenario

Fig. 2.   Cumulative distribution function of the mean normalized power of centralized log barrier technique
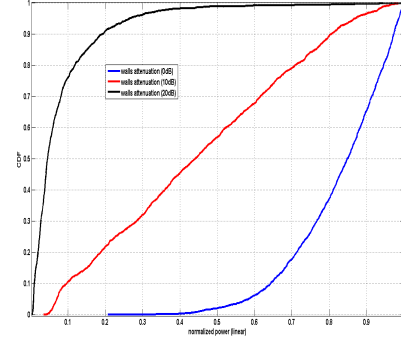

Fig. 3.   Cumulative distribution function of the mean normalized power of centralized log barrier technique (Dynamic case)
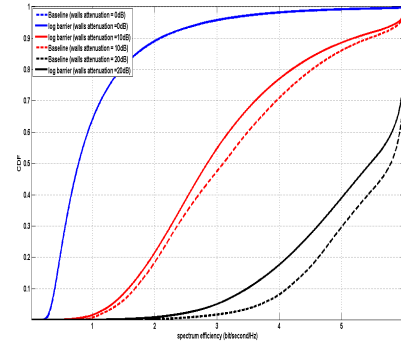

Fig. 4.   Cumulative distribution function of the capacity (Dynamic case)

1) Semi- static system level simulations where 57000 UE positions are uniformly sampling the coverage area of the femto base stations.
2) Dynamic system level simulations where independent Monte- Carlo simulation are performed for 100 snapshots of 570 random UE positions.

In the semi-static scenario, the performance of centralized log barrier energy saving considering *long term* coverage constraints for each cell. The dynamic system level simulation model is used to simulate the performance of short term UE-dependent energy saving. In Figure 1, a typical coverage map is shown for the semi-static system level simulation scenario with 0dB indoor walls attenuation with the neighborhood graph of the network defining the lists Ω_ℓ. The spectral efficiency is by definition taken as $\arg \max_R (1 - P_{out}(R))$, where $P_{out}(R)$ is the outage probability of the LTE 20MHz OFDM modulation on an indoor channel model, for a transmitted data rate $0 \le R \le 6$ bit/second/Hz. The OFDM channel model used in the simulations indoor hot spot channel model (InH) [7].

The centralized log barrier energy saving algorithm is simulated for 100 Monte-Carlo trials of the femto base stations

relative positions in the semi- static deployment scenario. Different walls attenuations were considered in the simulation and the cumulative distribution function (CDF) of the energy saving efficiency is shown in figure (2). The energy saving efficiency is evaluated as the average transmit power of the femto base stations, normalized by the maximum transmit power of a femto base station, i.e. $P_{\max} = 20$dBm. Two observations can be made from figure (2):

- The overall normalized energy saving gain of the centralized log barrier algorithm is 20% for a 0 dB wall attenuation and up to 90% for a 20 dB wall attenuation.
- The energy saving gain fluctuation with respect to the femto base stations positions is 27% for a 0 dB wall attenuation and up to 116 % for a 10 dB wall attenuation.

It is important to note that the results shown in Figure 2 are obtained for equal cell edge capacity with respect to the baseline femtocell scenario where all the nodes are transmitting at $P_{\max} = 20$dBm.

Then, the relative positions of the femto base stations are fixed and dynamic system level model is simulated. The average normalized energy saving gain is shown in Figure 3 for different wall attenuations, The corresponding capacity CDF curves are given in Figure 4. The overall energy saving gain of the log barrier algorithm is similar to the gain of the quasi static case as well as the cell edge performance. However
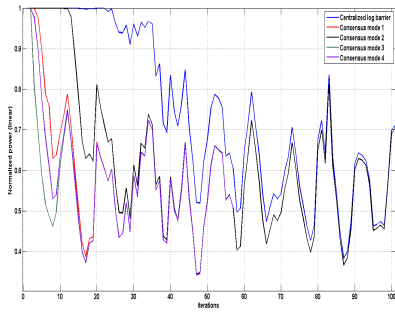
Fig. 5. Normalized mean power evolution with consensus (Dynamic case)

Fig. 6. Normalized mean power cumulative distributions for consensus and centralized energy saving (Dynamic case)

a degradation of 10% of the performance is observed in the higher capacity region for walls attenuation of 10 and 20dB. This is due to the fact that there is a probability to only have cell-center UEs in the cell that take less benefit from interference control.

Finally, random topology variations are introduced in the dynamic simulation scenario. This random topology change is modeled as random switching off/on process of a subset of the 25 femto base stations each 5 iterations of the log barrier method. The four consensus modes described in section (III) are simulated such that each femto base station cooperate with the femto base stations that are present in the topology, i.e. subset of the cooperation graph of figure (1) and each cooperating femto base stations increases their parameter t along with consensus iterations. In Figure 5, the normalized power is plotted for the centralized log barrier and the different consensus algorithms during the log barrier iterations.

All the consensus modes converge asymptotically to the performance of the centralized log barrier algorithm. The consensus modes 3 and 4 show the fastest convergence (around 10 iterations). So, fully distributed log barrier energy saving technique can achieve the performance of centralized energy saving when topology variation is highly dynamical. If the maximum number of iterations is fixed to 20, the performance of the centralized log barrier technique do not converges to the correct energy saving efficiency while the proposed decentralized log barrier energy saving maintain good performance as shown in figure (6).

Comparison of the cumulative distribution functions of the different consensus modes shows that the consensus modes 3 and 4 provides the best performance. In terms of performance, the consensus mode 4 doubles the energy saving efficiency of the log barrier technique in dynamic topology. When compared to the fixed topology case, the consensus mode 4 shows performance improvement around 66%. It is observed that the overall performance of the dynamic topology case is better than the fixed topology because the average interference is lowered by the nodes switching on/off process. In summary, the distributed consensus based energy saving technique is an effective way to achieve high energy saving gain through log barrier iterations and consensus update rules for femto
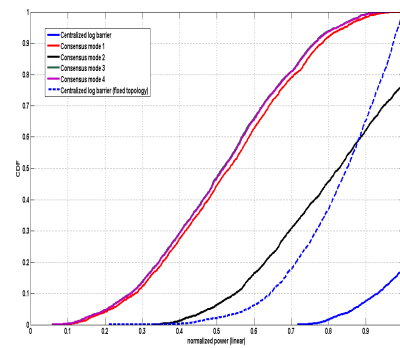
networks. The consensus modes 3 and 4 shows the best performance/convergence behavior.

V. CONCLUSION

In this paper we have proposed a decentralized energy saving technique based on the combination of consensus averaging and centralized log barrier interior point formulation of the optimization problem. We have considered four consensus iteration modes and compared the energy saving efficiency of the obtained algorithms with the baseline centralized log barrier power saving technique. Simulation results show that for long term coverage constraints, the centralized energy saving algorithm is feasible and achieves a worst case of 20% and maximum 30% of energy saving gain for 0dB walls attenuation and is dependent on the relative positions of the femto scenario. When the topology of the scenario changes during the log barrier iterations, the energy efficiency of the centralized log barrier technique is drastically reduced. Decentralized consenus-based log barrier technique can double the energy saving efficiency when consenus mode 4 is used. In future work we will investigate the extension of the proposed distributed energy saving techniques to macro base stations and heterogeneous networks deployments with larger number of nodes.

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