EM Channel Estimation for Coded OFDM Transmissions over Frequency-Selective Channel

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***Abstract*—We derive expectation-maximization (EM) based iterative algorithms to estimate the impulse response of multipath channel with coded OFDM system. We compare two ways for choosing EM complete data: a complete data built from observations and transmitted symbols (CL-EM) and complete data chosen by decomposing noise and observation components (NCD-EM). We also derive the Cramér-Rao lower bound (CRB) for coded OFDM transmission. Simulation results show that CL-EM has better convergence than NCD-EM and achieves the CRB.**

I. Introduction

Coded orthogonal frequency division multiplex (OFDM) has been chosen as the air interface for recent cellular and wireless LAN systems thanks to its good performance, its flexibility and its low implementation complexity. A number of channel estimation methods have been proposed for OFDM. When pilot symbols are available on some sub-carriers, initial estimates are easily obtained and can be improved through frequency- and time-domain interpolation [1] or according to the minimum mean square error (MMSE) criterion [2]. Blind channel estimation methods, not relying on the presence of pilot symbols, have also been proposed. For instance, they take benefit from the cyclostationarity of the OFDM signal [3]. However, all these algorithms have their limitations: lower accuracy, lower spectrum efficiency due to pilot overhead or higher sensitivity to Doppler effect in case of blind estimation.

As hardware capacity is continuously increasing, it becomes more feasible to implement iterative receivers allowing for improvement of the physical layer functions. Among them, channel estimation especially benefits from data aided methods requiring a feedback from the channel decoder. The iterative expectation-maximization (EM) [4] estimation algorithm is particularly well suited for OFDM systems with low pilot overhead. Instead of computing the maximum likelihood channel estimate from the observations only, it makes use of the so-called *complete data* Θ, which are not observed directly but only through incomplete data. Θ is a random variable. Thus, the log-likelihood can be averaged over Θ knowing the incomplete data and a current channel estimate. A new channel estimate is then obtained by maximizing the average log-likelihood, which results in the EM iterative structure. The likelihood increases along EM iterations [4]. A classic way (CL-EM) to choose the complete data is $\Theta = (\mathbf{X}, \mathbf{Y})$, where $\mathbf{X}$ is the transmitted signal and $\mathbf{Y}$ the observation [5]. In [6] and [7], for uncoded OFDM, complete data is obtained by decomposing the noise and observation components (NCD-EM). In [8], NCD-EM is applied to a coded single-carrier system.

In this paper, we compare CL-EM and NCD-EM for a coded OFDM system in terms of mean square error (MSE) and bit error rate (BER) performances. In addition, we derive the Cramér-Rao lower bound for this coded OFDM, as a reference for MSE performance.

II. System Description

We consider a coded OFDM signal transmitted over a single-input single-output (SISO) frequency-selective channel as shown in Fig. 1. An information binary sequence $\mathbf{S}$ is encoded into a coded sequence $\mathbf{C}$. The encoded bits are then interleaved by a pseudo-random interleaver and modulated. After pilot insertion, the obtained sequence $\mathbf{X} = [X_0, \cdots, X_{N-1}]^T$ is processed by an inverse fast Fourier transform (IFFT), which provides the time-domain sequence:

$$\mathbf{x} = [x_0, \cdots, x_{N-1}]^T = \mathbf{D}^H \mathbf{X} \tag{1}$$

where $\mathbf{D}$ is the normalized $N \times N$ FFT matrix. After insertion of a cyclic prefix (CP) with length L_{CP}, the transmitted OFDM symbol is $\mathbf{x}' = [x_{N-L_{CP}}, \cdots, x_{N-1}, \mathbf{x}^T]^T$. The received sequence is

$$y'_k = \sum_{l=0}^{L-1} h_l x'_{k-l} + n_k, \quad 0 \le k \le N + L_{CP} - 1, \tag{2}$$

where L is the channel length, $\mathbf{h} = [h_0, \cdots, h_{L-1}]^T$ is the channel impulse response, and n_k is a complex Gaussian noise with zero mean and variance $2\sigma^2$. After CP removal, the received time domain sequence $\mathbf{y} = [y_0, \cdots, y_{N-1}]^T$ is processed by FFT. The received sequence in the frequency domain is

$$\mathbf{Y} = [Y_0, \cdots, Y_{N-1}]^T = \mathbf{D}\mathbf{y}. \tag{3}$$

We assume that the impulse response is constant over one OFDM symbol. Thanks to OFDM modulation,

$$\mathbf{Y} = \text{diag}(\mathbf{H})\, \mathbf{X} + \mathbf{N} \tag{4}$$

where $\mathbf{N} = [N_0, \cdots, N_{N-1}]^T = \mathbf{D}\mathbf{n}$ has the same distribution as $\mathbf{n}$, and $\mathbf{H} = [H_0, \cdots, H_{N-1}]^T$ represents the channel frequency response:

$$\mathbf{H} = \mathbf{\Omega}\mathbf{h}, \tag{5}$$

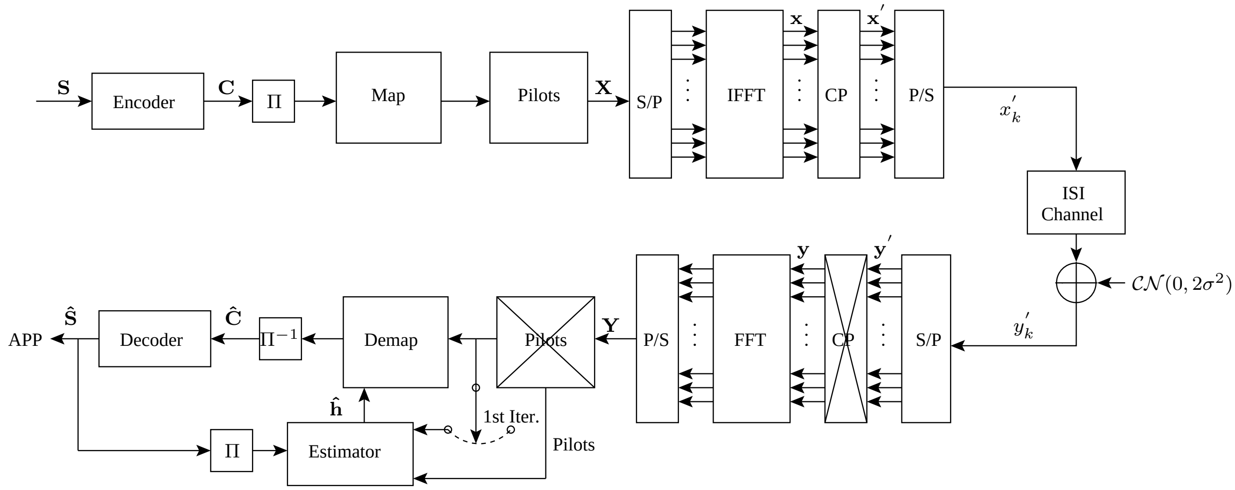

Fig. 1. Coded OFDM system model with proposed EM based channel estimator. The initial estimate is obtained from pilot symbols only, whereas the estimates in following iterations are obtained from both pilot and data symbols.

where the $N \times L$ matrix $\mathbf{\Omega}$ is built from the L first columns of $\sqrt{N}\mathbf{D}$. In (4), diag $(\mathbf{H})$ represents a diagonal matrix with H_k as its (k, k) entry.

An initial channel estimate is obtained from pilots included in sequence $\mathbf{Y}$. Data symbols are demodulated, and the obtained soft information on coded bits is de-interleaved into sequence $\hat{\mathbf{C}}$, which is processed by the decoder. After decoding, the a posteriori probabilities are fed back to the EM-based estimator which produces a new channel estimate for next demodulation and decoding. Thus, the soft information and the channel estimate will be improved along iterations.

III. EM-BASED CHANNEL ESTIMATION

A. Notations for the EM Algorithm

The EM algorithm provides a recursive solution to ML estimation [4] [9] and it performs a two-step procedure:

1) E-step: compute the auxiliary function $Q\left(\mathbf{h}|\mathbf{h}^{(i)}\right) = E_{\Theta}\left[\log p\left(\Theta\left|\mathbf{h}\right.\right)|\mathbf{Y}, \mathbf{h}^{(i)}\right]$;
2) M-step: update the parameters $\mathbf{h}^{(i+1)} = \arg\max_{\mathbf{h}} Q\left(\mathbf{h}|\mathbf{h}^{(i)}\right)$,

where Θ is called the complete data. The way to choose the complete data is one of the issues to be solved. A classic way is to choose $\Theta = (\mathbf{X}, \mathbf{Y})$. We call it classic EM (CL-EM). For superimposed signals, it has been shown in [10] that the complete data can be chosen by decomposing the noise components into independent noise processes. We call it noise component decomposition EM (NCD-EM). NCD-EM has been utilized in some applications, such as uncoded OFDM with single and multiple transmit antennas ([7] and [6] respectively) and coded single-carrier transmission on fading channel with uncorrelated and correlated paths ([8] and [11] respectively).

B. Classic EM Channel Estimation

Here, we choose the complete data with the classic method [5]:

$$\text{Complete data } \Theta = (\mathbf{X}, \mathbf{Y}).$$

$\mathbf{X}$ is called the missing data and $\mathbf{Y}$ is called the incomplete data. So, the E-step can be re-written as

$$Q\left(\mathbf{h}|\mathbf{h}^{(i)}\right) = E_{\mathbf{X}}\left[\log p\left(\mathbf{X}, \mathbf{Y}\left|\mathbf{h}\right.\right)|\mathbf{Y}, \mathbf{h}^{(i)}\right]. \tag{6}$$

If all values of $\mathbf{X}$ are equi-probable, the auxiliary function is

$$Q\left(\mathbf{h}|\mathbf{h}^{(i)}\right) = \sum_{\mathbf{X}} \log p\left(\mathbf{Y}\left|\mathbf{X}, \mathbf{h}\right.\right) APP_i\left(\mathbf{X}\right), \tag{7}$$

where $APP_i\left(\mathbf{X}\right) = P\left[\mathbf{X}|\mathbf{Y}, \mathbf{h}^{(i)}\right]$ is the a posteriori probability of $\mathbf{X}$.

1) EM with Energy Constraint: In order to estimate the channel impulse response $\mathbf{h}$, we re-write (4) as

$$\mathbf{Y} = \text{diag}\left(\mathbf{X}\right)\mathbf{\Omega}\mathbf{h} + \mathbf{N}. \tag{8}$$

With the probability density function (pdf) of $\mathbf{Y}$ given $\mathbf{X}$ and $\mathbf{h}$, the auxiliary function can be written as

$$\begin{aligned} Q\left(\mathbf{h}|\mathbf{h}^{(i)}\right) = &-\frac{1}{2\sigma^2}\sum_{X} \parallel \mathbf{Y} - \text{diag}\left(\mathbf{X}\right)\mathbf{\Omega}\mathbf{h} \parallel^2 \, APP_i\left(\mathbf{X}\right) \\ &- \sum_{\mathbf{X}} N \, \log 2\pi\sigma^2 APP_i\left(\mathbf{X}\right). \end{aligned} \tag{9}$$

In time varying multi-path Rayleigh channel, the channel coefficients change from one OFDM symbol to another. However, for each OFDM symbol we can define a real positive number C such that:

$$\parallel \mathbf{h} \parallel^2 = C. \tag{10}$$

In order to update the parameters with EM algorithm (M-step), taking into account (10), we utilise the Lagrange multipliers:

$$\begin{cases} \dfrac{\partial}{\partial \mathbf{h}} Q\left(\mathbf{h}|\mathbf{h}^{(i)}\right) + \mathbf{\Lambda}\dfrac{\partial}{\partial \mathbf{h}} g\left(\mathbf{h}\right) = 0 \\ g\left(\mathbf{h}\right) = 0 \end{cases} \tag{11}$$

where $\mathbf{\Lambda} = \text{diag}\left([\lambda, \cdots, \lambda]^T\right)$, and

$$g(\mathbf{h}) = \| \mathbf{h} \|^2 - C. \tag{12}$$

From (9), we have

$$\frac{\partial}{\partial \mathbf{h}} Q\left(\mathbf{h}|\mathbf{h}^{(i)}\right) = \frac{1}{2\sigma^2} \sum_{\mathbf{X}} APP_i(\mathbf{X})\, \mathbf{\Omega}^T \text{diag}(\mathbf{X})^T \mathbf{Y}^* - \frac{1}{2\sigma^2} \sum_{\mathbf{X}} APP_i(\mathbf{X})\, \mathbf{\Omega}^T \text{diag}(\mathbf{X})^T \text{diag}(\mathbf{X})^* \mathbf{\Omega}^* \mathbf{h}^*, \tag{13}$$

where $(\cdot)^*$ stands for complex conjugation and $(\cdot)^T$ stands for transpose. From (12), we also have

$$\frac{\partial}{\partial \mathbf{h}} g(\mathbf{h}) = \mathbf{h}^*. \tag{14}$$

Substituting (13) and (14) into the first equation of (11), the new channel parameters $\mathbf{h}^{(i+1)}$ can be calculated as:

$$\mathbf{h}^{(i+1)} = \left(\mathbf{\Omega}^H \mathbf{R}^*_{N\times N} \mathbf{\Omega} - \mathbf{\Lambda}'\right)^{-1} \mathbf{\Omega}^H \widetilde{\text{diag}(\mathbf{X})}^H \mathbf{Y}, \tag{15}$$

where $(\cdot)^H$ stands for transpose-conjugate and $\mathbf{\Lambda}' = 2\sigma^2 \mathbf{\Lambda}$; $\widetilde{\text{diag}(\mathbf{X})}^T$ is the $N \times N$ diagonal matrix of soft estimates of $\mathbf{X}$:

$$\widetilde{\text{diag}(\mathbf{X})} \triangleq \sum_{\mathbf{X}} APP_i(\mathbf{X}) \,\text{diag}(\mathbf{X}); \tag{16}$$

$\mathbf{R}_{N\times N}$ is a $N \times N$ matrix:

$$\mathbf{R}_{N\times N} = \sum_{\mathbf{X}} APP_i(\mathbf{X}) \,\text{diag}(\mathbf{X})^T \text{diag}(\mathbf{X})^*. \tag{17}$$

We take advantage of iterative estimation to get the value of $\mathbf{\Lambda}'$. By using the first equation of (11), (13) and (14), we have:

$$\mathbf{\Lambda}' \mathbf{h}^* = \mathbf{\Omega}^T \mathbf{R}_{N\times N} \mathbf{\Omega}^* \mathbf{h}^* - \mathbf{\Omega}^T \widetilde{\text{diag}(\mathbf{X})}^T \mathbf{Y}^*. \tag{18}$$

In (18), we replace $\mathbf{h}$ with $\mathbf{h}^{(i)}$ to calculate $\mathbf{\Lambda}'$, i.e.,

$$\mathbf{\Lambda}' \mathbf{h}^{(i)*} = \mathbf{\Omega}^T \mathbf{R}_{N\times N} \mathbf{\Omega}^* \mathbf{h}^{(i)*} - \mathbf{\Omega}^T \widetilde{\text{diag}(\mathbf{X})}^T \mathbf{Y}^* \triangleq \mathbf{V}. \tag{19}$$

By making use of $\|\mathbf{h}\|^2 = C$, we obtain the value of $\mathbf{\Lambda}'$:

$$\mathbf{\Lambda}' = \frac{1}{C} \mathbf{h}^{(i)T} \mathbf{V}. \tag{20}$$

In addition, λ' is a real number, so,

$$\lambda' = \Re e\left\{\frac{1}{C} \sum_{l=1}^{L} h_l^{(i)} v_l\right\}. \tag{21}$$

The value of C for each OFDM symbol can be obtained from the initial estimation based on pilots:

$$C = \| \mathbf{h}^{(0)} \|^2. \tag{22}$$

For a phase modulated system, all symbols have the same power $\mathcal{A}$, and (17) can be simplified to:

$$\mathbf{R}_{N\times N} = \mathcal{A} \mathbf{I}_{N\times N}. \tag{23}$$

In addition, from the orthogonality of $\mathbf{\Omega}$, we have:

$$\mathbf{\Omega}^H \mathbf{\Omega} = N \mathbf{I}_{L\times L}. \tag{24}$$

Using (23) and (24), (15) can be simplified to:

$$\mathbf{h}^{(i+1)} = \frac{1}{\mathcal{A}N - \lambda'} \mathbf{\Omega}^H \widetilde{\text{diag}(\mathbf{X})}^H \mathbf{Y}. \tag{25}$$

2) EM without Energy Constraint : Without considering the condition $\| \mathbf{h} \|^2$, we only utilise the auxiliary function and make $\frac{\partial}{\partial \mathbf{h}} Q\left(\mathbf{h}|\mathbf{h}^{(i)}\right) = 0$ to obtain the new channel coefficients in M-step. By making use of (13), we have

$$\mathbf{h}^{(i+1)} = \left(\mathbf{\Omega}^H \mathbf{R}^*_{N\times N} \mathbf{\Omega}\right)^{-1} \mathbf{\Omega}^H \widetilde{\text{diag}(\mathbf{X})}^H \mathbf{Y}. \tag{26}$$

For phase modulated system, (26) can be simplified to:

$$\mathbf{h}^{(i+1)} = \frac{1}{\mathcal{A}N} \mathbf{\Omega}^H \widetilde{\text{diag}(\mathbf{X})}^H \mathbf{Y}. \tag{27}$$

C. NCD-EM Channel Estimation

In this section, NCD-EM algorithm is derived with the complete data chosen by decomposing the noise and observation components. NCD-EM channel estimation for an uncoded OFDM system has been given in [6] and the NCD-EM algorithm for coded OFDM system is derived here.

In order to estimate the channel impulse response $\mathbf{h}$, we re-write (4) as:

$$\mathbf{Y} = \mathbf{A}\mathbf{h} + \mathbf{N} = \sum_{l=0}^{L-1} \mathbf{A}_l h_l + \mathbf{N}, \tag{28}$$

where $\mathbf{A} = \text{diag}(\mathbf{X})\, \mathbf{\Omega}$. $\mathbf{A}_l$ is the lth column of matrix $\mathbf{A}$ and $[\mathbf{A}]_{l,k} = a_{l,k}$. From (28), we get

$$Y_k = \sum_{l=0}^{L-1} a_{l,k} h_l + N_k \qquad 0 \le k \le N-1. \tag{29}$$

The noise and observed data are decomposed as in [6]:

$$Z_{l,k} = a_{l,k} h_l + N_{l,k} \qquad 0 \le k \le N-1, \tag{30}$$

where $N_k = \sum_{l=0}^{L-1} N_{l,k}$. Thus,

$$Y_k = \sum_{l=0}^{L-1} Z_{l,k}. \tag{31}$$

We choose the complete data as $\Theta = (\mathbf{Z}, \mathbf{A})$ and we obtain the auxiliary function in (32) [8], where $\mathbf{a_k}$ is the kth row of the matrix $\mathbf{A}$, ς_l is the lth component of vector ς and

$$E\left\{Z_{l,k}|\mathbf{a_k} = \varsigma, \mathbf{Y}, \mathbf{h}^{(i)}\right\} = h_l^{(i)} \varsigma_l + \beta_l \left(Y_k - \sum_{l=0}^{L-1} h_l^{(i)} \varsigma_l\right) \tag{33}$$

is the conditional expectation of $Z_{l,k}$ given $\mathbf{a_k} = \varsigma$. β_l controls the rate of convergence of the EM algorithm [10]. We choose $\beta_l = \frac{1}{L}$ [8] [11] in our simulations.

From (28), $\mathbf{a_k} = X_k \mathbf{\Omega}_k$, where $\mathbf{\Omega}_k$ is the kth row of the matrix $\mathbf{\Omega}$. Hence, the a posteriori conditional probability in (32) can be written as:

$$\text{P}\left(\mathbf{a_k} = \varsigma|\mathbf{Y}, \mathbf{h}^{(i)}\right) = \text{P}\left(X_k = \alpha_m|\mathbf{Y}, \mathbf{h}^{(i)}\right), \tag{34}$$

where α_m represents the set of possible symbols in the mapping constellation. The a posteriori conditional probability is given by the decoder. Taking the partial derivative of (32) with respect to each channel coefficient h_l and making the derivative to be zero, the new channel coefficient estimates $h_l^{(i+1)}$ can be expressed as (35).

IV. CRAMER-RAO BOUND FOR CODED OFDM

The Cramér-Rao Bound (CRB) provides an MSE lower bound to evaluate how good an unbiased estimator can be [12] [13]. Besides, modified CRB (MCRB) is a looser bound assuming perfect knowledge of the transmitted signal [14] [15]. Its computation is less complex. The CRB for the uncoded OFDM system and the MCRB for the OFDM system have been given in [6]. Here, we derive the CRB for a coded OFDM system. For the vector parameter $\mathbf{h}$ [13]:

$$CRB\left(h_l\right) = I_{ll}^{-1}\left(\mathbf{h}\right) \ , \quad l = 0, \cdots, L-1 \tag{36}$$

where $I\left(\mathbf{h}\right)$ is the Fisher information matrix:

$$I\left(\mathbf{h}\right) = E_{\mathbf{Y}}\left\{\frac{\partial}{\partial \mathbf{h}}\log\, p\left(\mathbf{Y}|\mathbf{h}\right)\left(\frac{\partial}{\partial \mathbf{h}}\log\, p\left(\mathbf{Y}|\mathbf{h}\right)\right)^H\right\}. \tag{37}$$

For coded transmission, the log-likelihood function is:

$$\log\, p\left(\mathbf{Y}|\mathbf{h}\right) = \log\sum_i P\left[\mathbf{X} = \mathbf{M}_i\right] p\left(\mathbf{Y}|\mathbf{X} = \mathbf{M}_i,\, \mathbf{h}\right), \tag{38}$$

where i enumerates all possible values $\mathbf{M}_i$ of $\mathbf{X}$. Differentiating (38) and making use of Bayes' rule, we obtain [16]

$$\begin{aligned}&\frac{\partial}{\partial \mathbf{h}}\log\, p\left(\mathbf{Y}|\mathbf{h}\right)\\ &= \sum_i APP\left(\mathbf{X} = \mathbf{M}_i\right)\ \frac{\partial}{\partial \mathbf{h}}\log\, p\left(\mathbf{Y}|\mathbf{X} = \mathbf{M}_i,\, \mathbf{h}\right).\end{aligned} \tag{39}$$

With the probability density function (pdf) of $\mathbf{Y}$ given $\mathbf{X}$ and $\mathbf{h}$, we also have

$$\begin{aligned}\frac{\partial}{\partial \mathbf{h}}\log\, p\left(\mathbf{Y}|\mathbf{X} = \mathbf{M}_i,\, \mathbf{h}\right) &= -\frac{1}{2\sigma^2}\mathbf{\Omega}^T\mathrm{diag}\left(\mathbf{M}_i\right)^T\mathbf{Y}^*\\ &+ \frac{1}{2\sigma^2}\mathbf{\Omega}^T\mathrm{diag}\left(\mathbf{M}_i\right)^T\mathrm{diag}\left(\mathbf{M}_i\right)^*\mathbf{\Omega}^*\mathbf{h}^*.\end{aligned} \tag{40}$$

Substituting (40) into (39), we obtain

$$\begin{aligned}&\frac{\partial}{\partial \mathbf{h}}\log\, p\left(\mathbf{Y}|\mathbf{h}\right)\\ &= -\frac{1}{2\sigma^2}\mathbf{\Omega}^T\widetilde{\mathrm{diag}\left(\mathbf{X}\right)}^T\mathbf{Y}^* + \frac{1}{2\sigma^2}\mathbf{\Omega}^T\mathbf{R}_{N\times N}\mathbf{\Omega}^*\mathbf{h}^*.\end{aligned} \tag{41}$$

Substituting (41) into (37), we obtain

$$CRB\left(\mathbf{h}\right) = \sum_{l=0}^{L-1} CRB\left(h_l\right) = \mathrm{trace}\left(I^{-1}\left(\mathbf{h}\right)\right). \tag{42}$$

V. SIMULATION AND DISCUSSION

We show some simulation results of the coded OFDM system introduced in section II for an ISI Rayleigh channel with 6-tap rectangular impulse response. Each OFDM symbol is comprised of 128 sub-carriers and the CP length is 16 samples. We use a half rate 64-state (133, 171) convolutional code and 16-QAM modulation. The pseudo-random interleaver size is 480 and 8 pilot symbols are uniformly inserted among data sub-carriers. Figure 2 shows CRBs for OFDM systems without coding and with two different coding schemes (1/2-rate and 1/5-rate). The better the encoding, the lower the CRB. For high E_s/N_0, all CRBs converge to the MCRB as APP information from the decoder becomes perfect; for low E_s/N_0, CRBs for the coded systems converge to the CRB for the uncoded system as decoding does not bring improvement anymore.

We first compare CL-EM with energy constraint (EC-EM) and CL-EM without energy constraint (CL-EM). From Fig. 3, we observe that, for both of them, the MSE already converges at the second iteration at high E_s/N_0. However, EC-EM does not reach the CRB, whereas CL-EM does. Indeed, for EC-EM, the estimate of C is not accurate enough and degrades the channel estimation. From Fig. 4, we observe that the MSE degradation does not result in strong BER performance

$$\begin{aligned}Q\left(\mathbf{h}|\mathbf{h}^{(i)}\right) = &-\frac{1}{2\sigma^2}\sum_{l=0}^{L-1}|h_l|^2\sum_{k=0}^{N-1}\sum_{\varsigma}|\varsigma_l|^2\mathrm{P}\left(\mathbf{a_k} = \varsigma|\mathbf{Y}, \mathbf{h}^{(i)}\right)\\ &+ \frac{1}{\sigma^2}\sum_{l=0}^{L-1}\sum_{k=0}^{N-1}\Re e\left\{h_l^*\sum_{\varsigma}\varsigma_l^* E\left\{Z_{l,k}|\mathbf{a_k} = \varsigma, \mathbf{Y}, \mathbf{h}^{(i)}\right\}\mathrm{P}\left(\mathbf{a_k} = \varsigma|\mathbf{Y}, \mathbf{h}^{(i)}\right)\right\}\end{aligned} \tag{32}$$

$$h_l^{(i+1)} = \frac{\sum_{k=0}^{N-1}\sum_{\alpha_m}\alpha_m^* e^{j2\pi\frac{(l-1)(k-1)}{N}} E\left\{Z_{l,k}|X_k = \alpha_m, \mathbf{Y}, \mathbf{h}^{(i)}\right\}\mathrm{P}\left(X_k = \alpha_m|\mathbf{Y}, \mathbf{h}^{(i)}\right)}{\sum_{k=0}^{N-1}\sum_{\alpha_m}|\alpha_m|^2\mathrm{P}\left(X_k = \alpha_m|\mathbf{Y}, \mathbf{h}^{(i)}\right)} \tag{35}$$

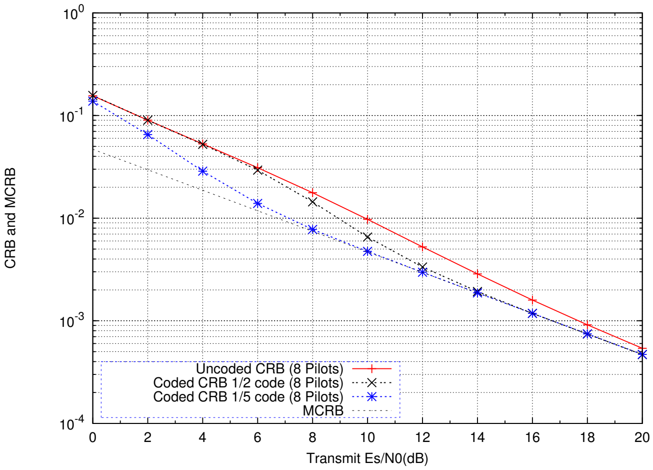

Fig. 2. Comparison of CRBs and MCRB.

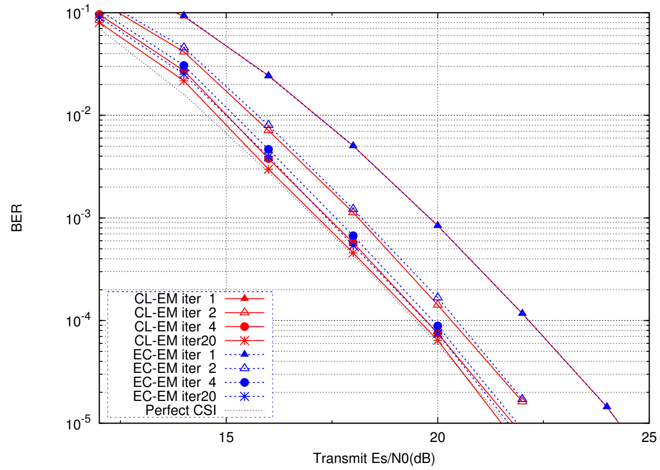

Fig. 4. Bit error rate (BER) performances of CL-EM and EC-EM.

degradation. However, since EC-EM is more complex than CL-EM, the latter is more efficient.

We also compare CL-EM and NCD-EM. From Fig. 5, we see that NCD-EM has much slower convergence than CL-EM, since it does not achieve CRB before the 16-th iteration. Similar behavior is observed in Fig. 6 for BER performance, whereas CL-EM approaches performance with perfect CSI only with 4 iterations.

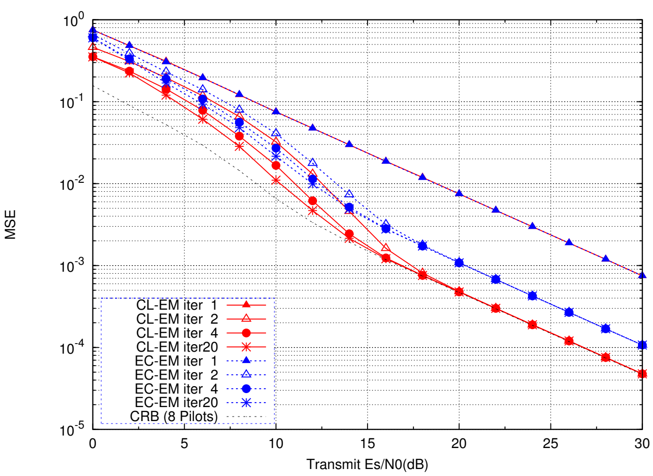

Fig. 3. Mean square error (MSE) of CL-EM and EC-EM.

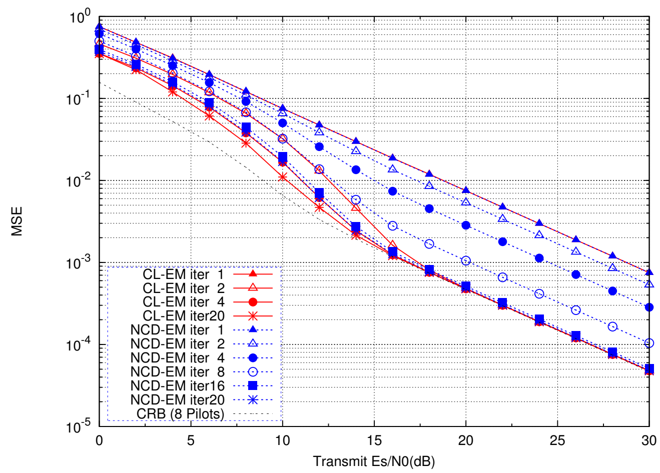

Fig. 5. Mean square error (MSE) performances of CL-EM and NCD-EM.

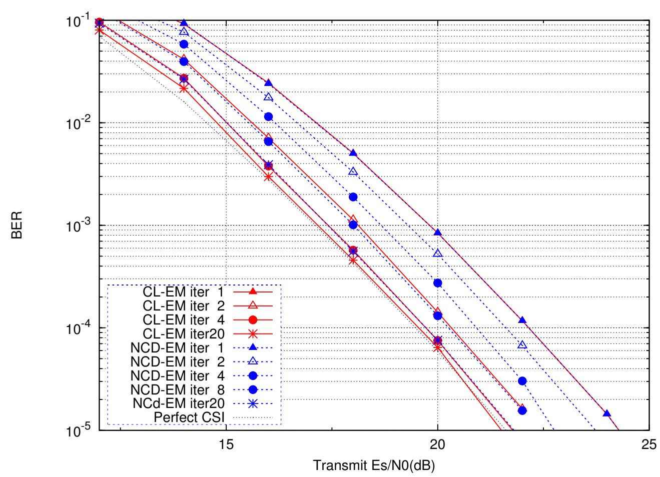

Fig. 6. Bit error rate (BER) performances of CL-EM and NCD-EM.

VI. CONCLUSION

We derived EM based iterative algorithms to estimate the channel impulse response for coded OFDM system. We compared CL-EM algorithm with the complete data $\Theta = (\mathbf{X}, \mathbf{Y})$ with a NCD-EM algorithm decomposing the noise and observation components to obtain the complete data. The CL-EM algorithm has faster convergence than NCD-EM. In addition, constraining the CL-EM algorithm with an initial estimate of the total impulse response power does not improve CL-EM performance due to the inaccuracy of this initial estimate.

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