Joint Channel Estimation and Decoding Using Gaussian Approximation in a Factor Graph over Multipath Channel

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***Abstract*—Joint channel estimation and decoding using belief propagation on factor graphs requires the quantization of probability densities since continuous parameters are involved. We propose to replace these densities by standard messages where the channel estimate is accurately modeled as a Gaussian mixture over multipah channel. Upward messages include symbol extrinsic information and downward messages carry mean values and variances for the Gaussian modeled channel estimate. Such unquantized message propagation leads to a complexity reduction and a performance improvement. Over multipath channel, the proposed belief propagation almost achieves the performance of iterative APP equalizer and outperforms MMSE equalizer.**

I. Introduction

Propagating messages in a suitable factor graph [1] is a systematic tool for deriving iterative algorithms. Among various receiver issues solved using the belief propagation algorithm (BP), also called sum-product algorithm [2], we can cite decoding, channel estimation, synchronization and detection [3]. [4] presents a BP handling continuous variables, in which canonical distributions are used for quantizing probability distributions, in order to propagate discrete probability distributions. However, the degree of quantization has a strong impact on estimation accuracy and performance. Even adapting the quantization in each iteration of BP, as proposed in [5] and [6], does not fill the complexity gap between BP and other algorithms. Instead of relying on quantization, we propose here to model probability distributions as mixtures of Gaussian distributions. It allows for estimation improvement and complexity reduction simultaneously. The BP with Gaussian approximation over single path channel has been presented in [7]. However, in many practical communication systems, symbols are transmitted over a channel with intersymbol interference (ISI). In this paper, we focus on BP with Gaussian approximation over multipath channel.

Over ISI channel, received symbols are usually processed by an equalizer in the receiver. A number of important equalizers have been presented in former works, including iterative a posteriori probability (APP) equalizer [8] and minimum mean square error (MMSE) equalizer [9]; however, all these equalizers have to work together with a channel estimator to obtain the channel coefficients. A factor graph with BP can help defining the iterative receiver in a systematic way and implementing joint channel estimation and decoding. However, the quantization method will make BP unfeasible over multipath channel. Thus, the proposed Gaussian approximation is required.

The paper is structured as follows. Section II explains how the transmission system is modeled using a factor graph and how BP is applied. Section III presents the approximation of the distribution of channel estimates over multipath channel in BP by a mixture of Gaussian distributions. In Section IV, APPs are computed from the approximated distribution. Continuous upward messages in the factor graph are presented in Section V. The paper ends with simulation results in Section VI.

In the sequel, messages that are not based on quantized densities will be referred as continuous messages.

II. System Model and Factor Graph

We consider a coded system with transmission over a single-input single-output (SISO) multi-path channel as shown in Fig. 1. An information binary sequence b_i is encoded,

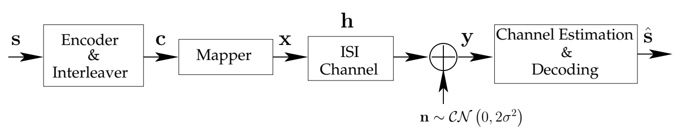

Fig. 1. System model.

interleaved and modulated into N BPSK symbols x_k. After convolution with an impulse response made of L taps, i.e., L complex Gaussian channel coefficients $h_l \sim \mathcal{CN}(0, 1/L)$, and addition of a complex Gaussian noise $n_k \sim \mathcal{CN}\left(0, 2\sigma_n^2\right)$, the channel outputs y_k are processed by a receiver performing joint channel estimation and decoding. Finally, the receiver outputs the estimated information sequence $\hat{b}_i$. The system model is described by

$$y_k = \sum_{l=0}^{L-1} h_l \, x_{k-l} + n_k, \qquad 0 \leq k \leq N-1. \tag{1}$$

We re-write (1) in matrix form:

$$y_k = \mathbf{x}_k^{\mathrm{T}} \mathbf{h} + n_k, \tag{2}$$

where $\mathbf{x}_k = (x_k, x_{k-1}, \cdots, x_{k-L+1})^{\mathrm{T}}$ represents the symbol vector at time instant k and $\mathbf{h} = (h_0, \cdots, h_{L-1})^{\mathrm{T}}$ represents the ISI channel.

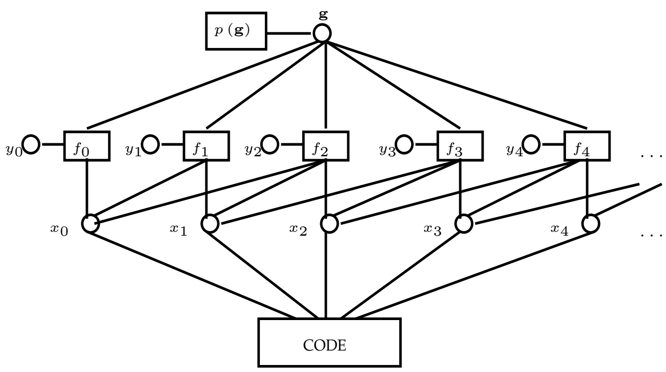

Fig. 2. Factor graph for multipath channel.

The corresponding factor graph for 3 taps is built in Fig. 2 following [4], where $\mathbf{g}$ is a quantized estimate of $\mathbf{h}$ and $p(\mathbf{g})$ represents the quantized distribution of the known a priori of $\mathbf{g}$. In this paper, we take the estimate from pilots as the a priori of $\mathbf{g}$:

$$p(\mathbf{g}) \triangleq p_{\mathrm{p}}(\mathbf{g}) = \prod_{k=0}^{L_{\mathrm{p}}-1} \mu_{f_{\mathrm{p},k}\to\mathbf{g}}, \tag{3}$$

as shown in Fig. 3(a). For upward messages, in node CODE, a forward-backward algorithm computes the extrinsic information for each deinterleaved coded bit. Taking interleaving into account, the extrinsic information ex_k is propagated to nodes x_k. In node x_k, the message $\mu_{x_k\to f_{k+l}} = \xi_{k,l}$ is obtained by multiplying all messages into x_k:

$$\mu_{x_k\to f_{k+l}} = \xi_{k,l} = \mathrm{ex}_k \prod_{\substack{i=0\\ i\neq l}}^{L-1} \mu_{f_{k+i}\to x_k}, \tag{4}$$

as shown in Fig. 3(b). From each node f_k to node $\mathbf{g}$, a discrete distribution $\mu_{f_k\to\mathbf{g}}$ of the quantized estimate of $\mathbf{h}$ is computed and propagated based on a marginalization of the likelihood $p(y_k|\mathbf{x}_k, \mathbf{g})$ with respect to the transmitted symbol $\mathbf{x}_k$, as shown in Fig. 3(c).

For downward messages, the message $\mu_{\mathbf{g}\to f_k}$ is calculated as shown in Fig. 3(d):

$$\mu_{\mathbf{g}\to f_k} = p_{\mathrm{p}}(\mathbf{g}) \prod_{\substack{i=0\\ i\neq k}}^{N-1} \mu_{f_i\to\mathbf{g}}. \tag{5}$$

By multiplying message $\mu_{\mathbf{g}\to f_k}$ and all messages from $\mathbf{x}_{k,l} = (x_k, x_{k-1}, \cdots, x_{k-l+1}, x_{k-l-1}, \cdots, x_{k-L+1})$ into f_k (Fig. 3(e)), the APP of each transmitted symbol x_{k-l} is computed, marginalizing the likelihood $p(y_k|\mathbf{x}_k, \mathbf{g})$ with respect to $\mathbf{g}$ and $\mathbf{x}_{k,l}$. The final APP of each coded bit $P(x_k)$ is obtained by multiplying all messages from node f_k to x_k (Fig. 3(f)) and then propagated to node CODE. The whole process of propagating upward and downward messages is then iterated.

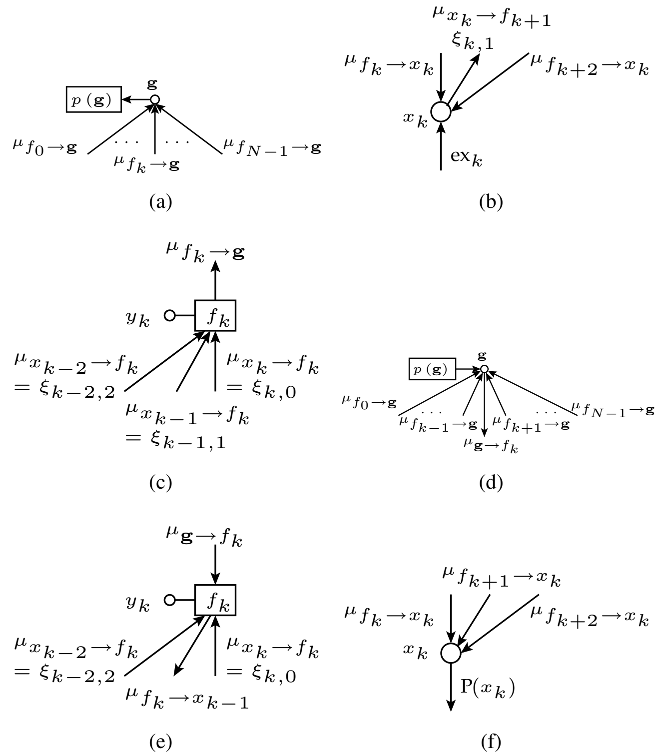

Fig. 3. Message propagation in factor graph.

III. Distribution of Channel Estimate

In the iterative receiver, initial estimate is obtained from known pilots and subsequent estimates from data symbols. Thus, the distribution of channel estimate will differ depending on the iteration.

A. Estimation based on pilots

L_{p} pilots $x_{\mathrm{p},k}$ $(0 \leqslant k \leqslant L_{\mathrm{p}}-1)$ are included in the transmitted sequence. From the L_{p} messages $\mu_{f_{\mathrm{p},k}\to\mathbf{g}}$ and (3), we get the discrete distribution of $\mathbf{g}$ [2]:

$$p_{\mathrm{p}}(\mathbf{g}) \propto \sum_{c=0}^{L^q-1} \delta(\mathbf{g}-\mathbf{g}_c) \prod_{k=0}^{L_{\mathrm{p}}-1} \exp\left(-\frac{|y_{\mathrm{p},k} - \mathbf{x}_{\mathrm{p},k}^{\mathrm{T}}\mathbf{g}_c|^2}{2\sigma_n^2}\right), \tag{6}$$

where

$$y_{\mathrm{p},k} = \sum_{l=0}^{L-1} h_l\, x_{\mathrm{p},k-l} + n_{\mathrm{p},k}; \tag{7}$$

$\mathbf{x}_{\mathrm{p,k}} = (x_{\mathrm{p},k}, \cdots, x_{\mathrm{p},k-L+1})^{\mathrm{T}}$; $\mathbf{g}_c = (g_0^c, \cdots, g_{L-1}^c)^{\mathrm{T}}$ is a quantization codebook of size L^q for channel estimate's probability density function (pdf) and $\delta(\cdot)$ denotes the Dirac delta function. Using a constant amplitude zero autocorrelation (CAZAC) sequence as pilots [10], (6) can be approximated as

$$p_{\mathrm{p}}(\mathbf{g}) \propto \sum_{c=0}^{L^q-1} \delta(\mathbf{g}-\mathbf{g}_c) \exp\left(-\frac{L_{\mathrm{p}}\mathcal{E}_{\mathrm{p}}}{2\sigma_n^2} |\mathbf{g}_c - \mathbf{h}|^2\right), \tag{8}$$

where $\mathcal{E}_{\mathrm{p}}$ represents the pilot energy. Hence, $p_{\mathrm{p}}(\mathbf{g})$ can be approximated as one Gaussian distribution $\mathcal{CN}(\mathbf{h}, \frac{2\sigma_n^2}{L_{\mathrm{p}}\mathcal{E}_{\mathrm{p}}})$.

B. Estimation based on data

Let $\mathbf{s}_{i,m} = (s_{i,0,m}, s_{i,1,m}, \cdots, s_{i,L-1,m})^{\mathrm{T}}$, where $0 \leq m \leq 2^L - 1$, represent the mth possible symbol vector and $\xi_{i-1,l}^m$ represent the probability that $x_{i-l} = s_{i,l,m}$. The product $\prod_{\substack{i=0\\i\neq k}}^{N-1} \mu_{f_i \to \mathbf{g}}$ can be expressed as $p_{\mathrm{d},k}(\mathbf{g})$ [2]:

$$
\begin{aligned}
p_{\mathrm{d},k}(\mathbf{g}) &\propto \prod_{\substack{i=0\\i\neq k}}^{N-1} \mu_{f_i\to\mathbf{g}} \propto \sum_{c=0}^{L^q-1} \delta(\mathbf{g}-\mathbf{g}_c) \times \\
&\prod_{\substack{i=0\\i\neq k}}^{N-1} \left\{ \sum_{m=0}^{2^L-1} \exp\left(-\frac{|y_i - \mathbf{s}_{i,m}^{\mathrm{T}}\mathbf{g}_c|^2}{2\sigma_n^2} \right) \prod_{l=0}^{L-1} \xi_{i-l,l}^m \right\} \\
&\propto \sum_{c=0}^{L^q-1} \delta(\mathbf{g}-\mathbf{g}_c) \times \\
&\sum_{j=0}^{2^{(N-1)L}-1} \exp\left(-\frac{1}{2\sigma_n^2} \sum_{\substack{i=0\\i\neq k}}^{N-1} |y_i - \mathbf{s}_i^{j\mathrm{T}}\mathbf{g}_c|^2 \right) \prod_{\substack{i=0\\i\neq k}}^{N-1} \prod_{l=0}^{L-1} \xi_{i-l,l}^j,
\end{aligned} \tag{9}
$$

where $\mathbf{s}_i^j = \left(s_{i,0}^j, s_{i,1}^j, \cdots, s_{i,L-1}^j\right)^{\mathrm{T}}$ is the value of symbol $\mathbf{x}_i$ in sequence j and $\xi_{i-l,l}^j$ is the probability that x_{i-l} equals $s_{i,l}^j$. After some calculations and approximations, (9) can be approximated as

$$
\begin{aligned}
p_{\mathrm{d},k}(\mathbf{g}) &\propto \sum_{c=0}^{L^q-1} \delta(\mathbf{g}-\mathbf{g}_c) \times \\
&\sum_{j=0}^{2^{(N-1)L}-1} \left\{ \prod_{l=0}^{L-1} \exp\left\{ -\frac{(N-1)}{2\sigma_n^2} \left| g_l^c - \frac{\left(U_l^j - V_l^j\right)}{(N-1)} h_l \right|^2 \right\} \times \right. \\
&\left. \exp\left\{ -\frac{(N-1)|h_l|^2}{2\sigma_n^2} \left[1 - \frac{\left(U_l^j - V_l^j\right)^2}{(N-1)^2} \right] \right\} \prod_{\substack{i=0\\i\neq k}}^{N-1} \xi_{i-l,l}^j \right\},
\end{aligned} \tag{10}
$$

where U_l^j (resp. V_l^j) is the number of items with $s_{i,l}^j x_{i-l}^* = +1$ (resp. $s_{i,l}^j x_{i-l}^* = -1$) in sequence j.

1) With high SNR, the decoder almost provides perfect extrinsic information. Thus, for a single sequence j with $U_l^j = N-1$, all $\xi_{i-l,l}^j \to 1$ and other terms are null:

$$
p_{\mathrm{d},k}(\mathbf{g}) \propto \sum_{c=0}^{L^q-1} \delta(\mathbf{g}-\mathbf{g}_c) \exp\left\{ -\frac{(N-1)}{2\sigma_n^2} |\mathbf{g}_c - \mathbf{h}|^2 \right\}; \tag{11}
$$

2) With low SNR, all probabilities $\xi_{i-l,l}^j$ are close to $1/2$ for BPSK modulation. In (10), there are $2^{(N-1)L}$ items. However, when considering the first exponential item in (10), only the sequences with $\left(U_l^j - V_l^j\right)^2 \geq (N-1)^2$, i.e., with $U_l^j = N-1$ or $V_l^j = N-1$, are not close to zero. Therefore, there are only 2^L dominant terms:

$$
\begin{aligned}
p_{\mathrm{d},k}(\mathbf{g}) \propto \sum_{c=0}^{L^q-1} \delta(\mathbf{g}-\mathbf{g}_c) \prod_{l=0}^{L-1} &\left\{ \beta_l \exp\left\{ -\frac{(N-1)}{2\sigma_n^2} |g_l^c - h_l|^2 \right\} \right. \\
&\left. + (1-\beta_l) \exp\left\{ -\frac{(N-1)}{2\sigma_n^2} |g_l^c + h_l|^2 \right\} \right\},
\end{aligned} \tag{12}
$$

where β_l denotes the normalized product of $\xi_{i-l,l}^j$; for a single channel tap, the distribution $p_{\mathrm{d},k}(g_l)$ is:

$$
\begin{aligned}
p_{\mathrm{d},k}(g_l) \propto \sum_{c=0}^{q-1} \delta(g_l - g_l^c) &\left\{ \beta_l \exp\left\{ -\frac{(N-1)}{2\sigma_n^2} |g_l^c - h_l|^2 \right\} \right. \\
&\left. + (1-\beta_l) \exp\left\{ -\frac{(N-1)}{2\sigma_n^2} |g_l^c + h_l|^2 \right\} \right\}.
\end{aligned} \tag{13}
$$

From (11), (12) and (13), for each channel tap, the pdf $p_{\mathrm{d},k}(g_l)$ can be approximated as a mixture of two Gaussian distributions; for the whole ISI channel, the pdf $p_{\mathrm{d},k}(\mathbf{g})$ can be approximated as a mixture of multiple Gaussian distributions which are the product of all pdfs of each tap with variance $\frac{2\sigma_n^2}{N-1}$.

IV. APP Evaluation from Downward Messages

With the conclusions in Section III, the known a priori discrete channel distribution $p_{\mathrm{p}}(\mathbf{g})$ can be approximated as one Gaussian distribution, and the discrete distribution of the product $\prod_{\substack{i=0\\i\neq k}}^{N-1} \mu_{f_i\to\mathbf{g}} = p_{\mathrm{d},k}(\mathbf{g})$ can be approximated as a mixture of multiple Gaussian distributions, where $p_{\mathrm{p}}(\mathbf{g})$ uses pilots and $p_{\mathrm{d},k}(\mathbf{g})$ uses the messages in the current iteration. Furthermore, together with the conclusion in [7], for each tap, there is always one dominant Gaussian distribution (with mean value h_l). Hence, when calculating APP, we consider only the dominant one ($\beta_l = 1$). Then, the discrete distributions of $p_{\mathrm{p}}(\mathbf{g})$ and $p_{\mathrm{d},k}(\mathbf{g})$ can both be reduced to L pairs of parameters: $(\hat{h}_{\mathrm{p},l}, \hat{\sigma}_{h\mathrm{p}}^2)$ for $p_{\mathrm{p}}(\mathbf{g})$ and $(\hat{h}_{\mathrm{d},k,l}, \hat{\sigma}_{h\mathrm{d}}^2)$ for $p_{\mathrm{d},k}(\mathbf{g})$. Thus, $p_{\mathrm{p}}(\mathbf{g})$ times $p_{\mathrm{d},k}(\mathbf{g})$ can also be approximated by a mixture of Gaussian distributions, i.e., the discrete distribution of message $\mu_{\mathbf{g}\to f_k}$ can be reduced to L pairs of parameters $(\hat{h}_{k,l}, \hat{\sigma}_h^2)$, denoted as $p_k(\mathbf{g})$. Obviously, $(\hat{h}_{k,l}, \hat{\sigma}_h^2)$ can be calculated from $(\hat{h}_{\mathrm{p},l}, \hat{\sigma}_{h\mathrm{p}}^2)$ and $(\hat{h}_{\mathrm{d},k,l}, \hat{\sigma}_{h\mathrm{d}}^2)$ that will be shown in the following part. Thus, we can calculate each downward message $\mu_{f_k \to x_{k-l}}$ in a continuous way, instead of computing it for each codebook value $\mathbf{g}_c$, and then marginalizing with respect to $\mathbf{g}$. It reduces the computation complexity.

With the discrete way, the probability of symbol vector $\mathbf{x}_k$ can be calculated as:

$$\mathrm{P}\left(\mathbf{x}_k=\mathbf{s}_{k,m}\right)=\sum_{c=0}^{L^q-1}\exp\left\{-\frac{1}{2\sigma_n^2}\left|y_k-\mathbf{s}_{k,m}^{\mathrm{T}}\mathbf{g}_c\right|^2\right\}\mathrm{P}\left(\mathbf{g}_c\right). \tag{14}$$

Equation (14) can be written in a continuous way as:

$$\mathrm{P}\left(\mathbf{x}_k=\mathbf{s}_{k,m}\right)=\underbrace{\int\cdots\int}_{L}\exp\left\{-\frac{1}{2\sigma_n^2}\left|y_k-\sum_{l=0}^{L-1}s_{k,l,m}g_l\right|^2\right\}\prod_{l=0}^{L-1}p_k\left(g_l\right)dg_0\cdots dg_{L-1}. \tag{15}$$

With some calculations, we have

$$\mathrm{P}\left(\mathbf{x}_k=\mathbf{s}_{k,m}\right)\propto\frac{1}{\hat{\sigma}_h^2\sum_{l=0}^{L-1}\left|s_{k,l,m}\right|^2+\sigma_n^2}\times\exp\left\{-\frac{\left|y_k-\mathbf{s}_{k,m}^{\mathrm{T}}\hat{\mathbf{h}}_k\right|^2}{2\left(\hat{\sigma}_h^2\sum_{l=0}^{L-1}\left|s_{k,l,m}\right|^2+\sigma_n^2\right)}\right\}, \tag{16}$$

where $\hat{\mathbf{h}}_k=\left(\hat{h}_{k,0},\cdots,\hat{h}_{k,L-1}\right)^{\mathrm{T}}$. According to Fig. 3(e) and Fig. 3(f), we have

$$\mathrm{P}\left(x_k=b_i\right)\propto\prod_{l=0}^{L-1}\sum_{s_{k+l,m,l}=b_i}\mathrm{P}\left(\mathbf{x}_{k+l}=\mathbf{s}_{k+l,m}\right)\prod_{\substack{l'=0\\l'\neq l}}^{L-1}\mu_{x_{k+l-l'}\to f_{k+l}}. \tag{17}$$

Thanks to the computation in (16), a single APP computation instead of L^q computations is performed for each symbol vector $\mathbf{x}_k$ with the Gaussian approximation. Thus, the global complexity is much reduced by the Gaussian approximation in the downward messages.

V. Estimation from Upward Messages

In order to improve the performance of the Gaussian approximation, we propose to increase the accuracy of $\hat{\mathbf{h}}_k$ using a continuous upward message.

Replacing the discrete distribution in (9) by an integral, we get continuous $p_{\mathrm{d},k}\left(\mathbf{g}\right)$:

$$p_{\mathrm{d},k}\left(\mathbf{g}\right)\propto\sum_{j=0}^{2^{(N-1)L}-1}\exp\left\{-\frac{1}{2\sigma_n^2}\sum_{\substack{i=0\\i\neq k}}^{N-1}\left|y_i-\mathbf{s}_i^{j\mathrm{T}}\mathbf{g}\right|^2\right\}\Delta_j, \tag{18}$$

where $\Delta_j=\prod_{\substack{i=0\\i\neq k}}^{N-1}\prod_{l=0}^{L-1}\xi_{i-l,l}^j$. Thus, we can get the distribution of g_l from (18):

$$p_{\mathrm{d},k}\left(g_l\right)\propto\underbrace{\int\cdots\int}_{\mathbf{g}'}p_{\mathrm{d},k}\left(\mathbf{g}\right)d\mathbf{g}', \tag{19}$$

where $\mathbf{g}'=\left(g_0,\cdots,g_{l-1},g_{l+1},\cdots,g_{L-1}\right)$. After some complex derivation and approximation, the pdf $p_{\mathrm{d},k}\left(g_l\right)$ can be approximately written as

$$p_{\mathrm{d},k}\left(g_l\right)\propto\sum_{j=0}^{2^{(N-1)L}-1}\Delta_j\exp\left\{-\frac{1}{2\sigma_n^2}\left|g_l\right|^2\Omega_{k,l}^j\right\}\times\exp\left\{\frac{1}{2\sigma_n^2}2\Re e\left\{g_l^*\Phi_{k,l}^j-g_l^*\sum_{\substack{l'=0\\l'\neq l}}^{L-1}\frac{\Phi_{k,l'}^j}{\Omega_{k,l'}^j}\mathrm{R}_{k,l',l}^j\right\}\right\}, \tag{20}$$

where $\Phi_{k,l}^j=\sum_{\substack{i=0\\i\neq k}}^{N-1}y_is_{i,l}^{j*}$, $\Omega_{k,l}^j=\sum_{\substack{i=0\\i\neq k}}^{N-1}\left|s_{i,l}^j\right|^2$ and $\mathrm{R}_{k,l',l}^j=\sum_{\substack{i=0\\i\neq k}}^{N-1}s_{i,l'}^js_{i,l}^{j*}$. Using (20) and considering normalization, we get

$$\hat{h}_{\mathrm{d},k,l}=\frac{\int_{g_l}g_lp_{\mathrm{d},k}\left(g_l\right)d\,g_l}{\int_{g_l}p_{\mathrm{d},k}\left(g_l\right)d\,g_l}\approx\frac{1}{\widetilde{\Omega}_{k,l}}\left(\widetilde{\Phi}_{k,l}-\sum_{\substack{l'=0\\l'\neq l}}^{L-1}\frac{\widetilde{\Phi}_{k,l'}}{\widetilde{\Omega}_{k,l'}}\widetilde{\mathrm{R}}_{k,l',l}\right), \tag{21}$$

where

$$\widetilde{\Phi}_{k,l}=\sum_{\substack{i=0\\i\neq k}}^{N-1}y_i\sum_m s_{i,l,m}^*\xi_{i,l}^m, \tag{22}$$

$$\widetilde{\mathrm{R}}_{k,l',l}=\sum_{\substack{i=0\\i\neq k}}^{N-1}\left(\sum_m s_{i,l',m}\xi_{i,l'}^m\right)\left(\sum_m s_{i,l,m}^*\xi_{i,l}^m\right) \tag{23}$$

and $\widetilde{\Omega}_{k,l}\approx(N-1)\mathcal{E}_{\mathrm{av}}$. Here, $\mathcal{E}_{\mathrm{av}}$ represents the average power of transmitted symbols.

Following the same steps, the estimation with a CAZAC pilot sequence can be calculated as

$$\hat{h}_{\mathrm{p},l}\approx\frac{1}{L_{\mathrm{p}}\mathcal{E}_{\mathrm{p}}}\sum_{k=0}^{L_{\mathrm{p}}-1}y_{\mathrm{p},k}x_{\mathrm{p},k-l}. \tag{24}$$

With (21), we can get $\hat{\mathbf{h}}_{\mathrm{d},k}=\left(\hat{h}_{\mathrm{d},k,0},\cdots,\hat{h}_{\mathrm{d},k,L-1}\right)^{\mathrm{T}}$ for $p_{\mathrm{d},k}\left(\mathbf{g}\right)$ by using the messages in current iteration; with (24), we can get $\hat{\mathbf{h}}_{\mathrm{p}} = \left(\hat{h}_{\mathrm{p},0}, \cdots, \hat{h}_{\mathrm{p},L-1}\right)^{\mathrm{T}}$. Together with $\hat{\sigma}^2_{h\mathrm{d}}$ and $\hat{\sigma}^2_{h\mathrm{p}}$, we can get

$$\hat{h}_{k,l} = \frac{\hat{\sigma}^2_{h\mathrm{p}} \hat{h}_{\mathrm{d},k,l} + \hat{\sigma}^2_{h\mathrm{d}} \hat{h}_{\mathrm{p},l}}{\hat{\sigma}^2_{h\mathrm{p}} + \hat{\sigma}^2_{h\mathrm{d}}} \quad \text{and} \quad \hat{\sigma}^2_h = \frac{\hat{\sigma}^2_{h\mathrm{p}} \hat{\sigma}^2_{h\mathrm{d}}}{\hat{\sigma}^2_{h\mathrm{p}} + \hat{\sigma}^2_{h\mathrm{d}}}, \tag{25}$$

where the value of $\hat{\sigma}^2_{h\mathrm{p}}$ and $\hat{\sigma}^2_{h\mathrm{d}}$ can be obtained from (8) and (10): $\frac{\sigma^2_n}{L_{\mathrm{p}}\mathcal{E}_{\mathrm{p}}}$ for pilot case and $\frac{\sigma^2_n}{N-1}$ for data case.

VI. Numerical results

Some simulation results are shown in this section for an ISI Rayleigh channel with 3-tap rectangular impulse response. We use a half rate 64-state (133, 171) convolutional code and BPSK modulation. The pseudo-random interleaver size is 1000. The number of pilots is 18.

The proposed BP with continuous downward and upward messages (BP-DUGA) is compared with iterative APP equalizer (APPEQ) and MMSE equalizer (MMSEEQ), both using EM channel estimation [11] [12]. From Fig. 4, we observe that the proposed BP-DUGA has better BER performance with perfect channel state information (PerCSI) than MMSE equalizer – about 1dB for 10^{-5} – where $K = 11$ represents the number of complex-valued tap weight coefficients of the equalizer. With 5 iterations, it also outperforms MMSEEQ+EM. From Fig. 5, we observe that BP-DUGA has BER performance very close to that of iterative APP equalizer with PerCSI and EM channel estimation. Using continuous downward and upward messages brings a complexity reduction compared to the quantization method.

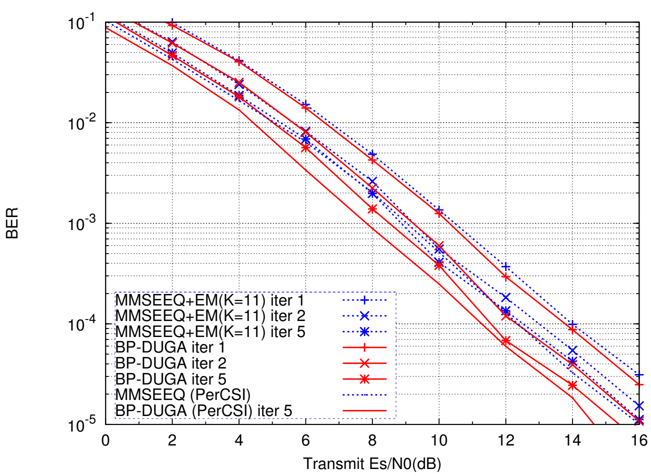

Fig. 4. Bit error rate performance comparison: BP-DUGA vs MMSE Equalizer with EM channel estimation.

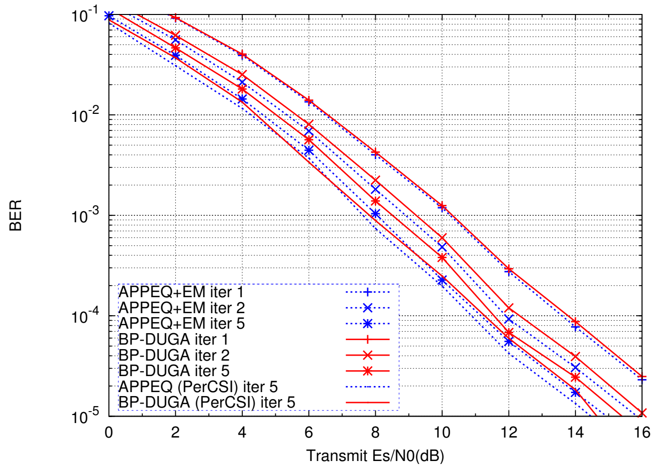

Fig. 5. Bit error rate performance comparison: BP-DUGA vs APP Equalizer with EM channel estimation.

VII. Conclusion

Thanks to an approximation of the distribution of the channel estimate as a mixture of Gaussian distributions, we improved the performance of BP and reduced its complexity by propagating continuous messages in the factor graph for multipath channel. The proposed BP-DUGA almost achieves the performance of the more complex APPEQ and outperforms MMSE equalizer. This paper is focusing on BPSK modulation. Nevertheless, the extension of the Gaussian approximation principle to a higher level modulation scheme is natural.

VIII. Acknowledgement

This work has been performed in the framework of the CELTIC project CP5-026 WINNER+.

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