Patched Distributed Space-Time Block Codes

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***Abstract*—In this paper, we propose Patched Distributed Space-Time Block Codes (DSTBC) for the Dynamic Decode and Forward (DDF) Relaying protocol, in which the relay transmits combination of information from the first and second phase of the DDF protocol, and the destination makes a linear combination of symbols received in the two phases. Thus, from the error correcting code point of view, the number of coded bits seeing a diversity-one channel reduces proportionnally as the number of symbols sent by the relay increases, which is not the case with known schemes. In other words, the length of the phase two of the DDF protocol needed to achieve full diversity can be reduced with respect to known schemes. Three examples of Patched DSTBC are presented, the Patched Alamouti, the Patched Golden Code and the Patched Silver Code whose outage analyses prove the diversity enhancement offered by the proposed scheme.**

I. Introduction:

Among the wide panel of studies on relaying, Dynamic Decode and Forward (DDF) protocols [1] have shown the great interest of taking the best benefit from the relay by limiting its use while ensuring an improvement of the performance [2], [3], [4], [5], [6]. The DDF protocol is composed of two phases, the first one when the relay and the destination listen to the source, ending when correct decoding occurs at the relay. The second phase follows during which the destination receives information from the source and the relay. Whatever happens at the relay, the transmission of a codeword ends as soon as the destination correctly decodes the information word.

All known DDF protocols, such as the ones using classical Distributed Space-Time Block Codes (DSTBC) [7] in the second phase of the DDF protocol, present the disadvantage that coded bits of the first phase are not protected by the relay-destination link. Thus, an equivalent Matryoshka block-fading channel [8] limits the diversity order observed after the error-correcting code decoding.

We propose that the relay transmits a combination of symbols from the first and second phase, so that a space-time code is built after a linear combination at the destination of the received symbols from the two phases. This allows for creating a better-conditionned equivalent block-fading channel, and for achieving full diversity in more cases.

In section II, the system model and transmission scheme are presented. in section III, we present a diversity analysis of the proposed Patched-DSTBC. In section IV, we present the definition of Patched-DSTBC and three pratical examples. In section V, we present outage analyses illustrating the gains brought by the Patched-DSTBC.

II. System Model and Transmission Scheme

We consider a source S, a relay R, carrying a single antenna, and a destination D with N_r reception antennas. We consider quasistatic flat-fading channels, i.e., the channel remains constant during at least the transmission of one codeword and is independant from one transmission to another. We note $\mathbf{h}_{SD}$, $\mathbf{h}_{SR}$ and $\mathbf{h}_{RD}$ the vectors of N_r complex gaussian distributed fading coefficients with zero mean and unit variance, of the source-destination, source-relay and relay-destination links, respectively. A complex gaussian noise of variance $2N_0$ is added at each receive antenna. Moreover, we note SNR_{SD}, SNR_{SR} and SNR_{RD} the signal to noise ratios of the source-destination, source-relay and relay-destination links, respectively.

A Bit interleaved Coded Modulation scheme (BICM) is used for transmission: a K-bit-long information word $\mathbf{b}$ is encoded by an error correcting code and interleaved to form a codeword $\mathbf{c}$; and modulated using a 2^{ms} QAM modulation.

A codeword $\mathbf{c}$ is segmented into two sub-frames of length L_1 and L_2 coded bits, each corresponding to the first and second phase of the DDF protocol and consequently depending on the correct decoding time at the relay. A Cyclic Redundancy Check (CRC) code is embedded into the information word in order for the destination and the relay to check if they correctly decode the message formed by successive concatenation of the received sub-frames. Note that $L_1 \geq K$ in order for the relay to be able to correctly decode the message at the end of the first sub-frame.

Let us assume that the relay R correctly decodes the information word after receiving the first sub-frame. By using the same encoding algorithm as the source, it can re-generate from the K correctly decoded information bits the symbols previously sent during the first sub-frame. Furthermore, it can generate symbols intended to be sent by the source S in the future, i.e., during the second sub-frame. Then, the relay R combines the symbols of the two sub-frames according to the Patched-DSTBC schemes proposed hereafter. Consequently, a subset of the symbols sent by the source during the first sub-frame are also transmitted through the relay-destination link. The relay sends the combinations of symbols on the same frequency-time ressource as the source. During the second sub-frame, the scheme obtained by the joint transmission from the source S and the relay R does not form itself a DSTBC. One novelty of our scheme is that it is needed to add the contribution (patch) of the symbols sent by the source during the first sub-frame to obtain the so-called Patched-DSTBC structure.

If the relay does not correctly decode the information word after receiving the first sub-frame sent by the source, the source sends the second sub-frame and the relay remains silent.

During the first sub-frame, a classical receiver is used at the destination D forming soft estimates of the coded bits in order to feed the inputs of a soft-input decoder. While the source is sending the second sub-frame of the codeword, the destination D receives the superimposition of symbols sent by the source S and the relay R. During the second sub-frame, the destination D converts the received space-time codewords resulting from the combination of symbols from the first and second phase into soft-bit estimates. We assume that the destination has the ability to detect if the relay is transmitting or not. This can be achieved for example by using pilot signals from the relay R.

A general definition of Patched-DSTBC and three practical examples are detailed in the following sections.

III. Diversity analysis of coded modulations with DSTBC and Patched-DSTBC protocols

When using a DSTBC, if the relay correctly decodes the message at the end of the first sub-frame, the L_1 coded bits of the first sub-frame are transmitted only through the source to destination data link, and their soft estimates at the input of the error correcting code decoder carry a diversity order N_r. The L_2 coded bits of the second sub-frame are transmitted on the source to destination data link and to the relay to destination link, and the structure of the DSTBC makes their soft estimates carry a diversity order $2N_r$. Thus, the error correcting code sees a block-fading channel with two blocks of size L_1 and L_2, respectively with diversity N_r and $2N_r$. The channel coefficient associated to the second block is a function of the channel coefficient associated to the first block. As a result, the error correcting code is transmitted over a Matryoshka block-fading channel [8] with parameters $\mathcal{M}(\{L_2, L_1\}, \{2N_r, N_r\})$. From [8], we can conclude that the full diversity order $2N_r$ is achieved after decoding only if $L_2 \geq K$. Fig.1a) illustrates an example of a DSTBC transmission using a distributed Alamouti scheme during the second sub-frame and the resulting Matryoshka channel.

The main idea of Patched-DSTBC is to virtually move some coded bits from the block of diversity N_r of the Matryoshka channel to the block of diversity $2N_r$. Let us assume that we combine at the receiver the received symbols of the second sub-frame with the first received symbols of the first sub-frame so that, in conjunction with the signals transmitted by the relay, a space-time code is built. Fig.1b) illustrates an example of Patched-DSTBC with the combination operation at the destination, and the resulting Matryoshka channel. Two scenarii of Patched-DSTBC are considered that result in different coding gains, as explained in next section.

In first scenario, called Patched-DSTBC with total use, the relay transmits during all symbols of the second sub-frame using the proposed Patched-DSTBC if $L_2 \leq L_1$. In that case, L_2 bits out of the L_1 coded bits of the first sub-frame now carry a diversity order $2N_r$. As a remark, if $L_2 = L_1$, all symbols sent by the source during the first sub-frame have been virtually moved to the full diversity block. Consequently, if $L_2 > L_1$, a Distributed Alamouti DDF (DA-DDF) [9] coding scheme can be used for all symbols further sent by the source and the relay. Therefore, the block-fading channel seen by the error correcting code is changed into a $\mathcal{M}(\{\min(2L_2, L_1+L_2), \max(L_1-L_2, 0)\}, \{2N_r, N_r\})$ Matryoshka channel. From [8], we can conclude that if this scenario of Patched-DSTBC is used, the full diversity order $2N_r$ is achieved after decoding only if $L_2 \geq K/2$ which allows for achieving the full diversity with less channel uses than with a DSTBC-DDF scheme. As a remark, if the relay does not correctly decode the message at the end of the first sub-frame, the diversity order observed at the output of the decoder is N_r.

In second scenario, called Patched-DSTBC with minimal use, the relay transmits using Patched-DSTBC for the minimal number of time periods that guarantees full diversity after decoding, and a DA-DDF scheme is used for the remaining time slots of the second sub-frame. Let us note T the number of bits the relay transmits using Patched-DSTBC, i.e., the number of bits moved from the block of diversity N_r to the block of diversity $2N_r$. The equivalent channel as seen by the decoder is a Matryoshka channel with parameters $\mathcal{M}(\{L_2+T, L_1-T\}, \{2N_r, N_r\})$. The relay only transmits $T = K - L_2$ in order to guarantee full diversity. As a remark, if $L_2 \geq K$, the relay transmits using DSTBC-DDF only.

In the next section, we propose a transmission scheme that meets the Patched-DSTBC requirements, and the performance of the two scenarios are studied in section V.

IV. Patched Distributed Space-Time Block Codes

Let us consider that the relay correctly decodes the message at the end of the first sub-frame. We consider a couple of time-slots of each sub-frame. The couple of time-slots of the first phase carries the symbols s_1 and s_2, and the received signal at the destination is $\mathbf{Y}_1$:

$$\mathbf{Y}_1 = \mathbf{h}_{SD}[s_1 \quad s_2] + \boldsymbol{\eta}_1 \tag{1}$$

where $\boldsymbol{\eta}_1$ is a $N_r \times 2$ white complex gaussian noise matrix with zero mean and variance $2N_0$ entries.

During the two time-slots of the second sub-frame, the source S transmits the symbols s_3 and s_4. We recall that, after correctly decoding the message, the relay can generate at the beginning of the second sub-frame former (e.g., s_1 and s_2) and future (e.g., s_3 and s_4) symbols sent by the source. Thus, the relay transmits two functions of the four aforementionned symbols: $f_1(s_1, s_2, s_3, s_4)$ and $f_2(s_1, s_2, s_3, s_4)$. Consequently, during these two time-slots of the second sub-frame, the destination will receive $\mathbf{Y}_2$:

$$\mathbf{Y}_2 = \mathbf{H}\begin{bmatrix} s_3 & s_4 \\ f_1(s_1, s_2, s_3, s_4) & f_2(s_1, s_2, s_3, s_4) \end{bmatrix} + \boldsymbol{\eta}_2 \tag{2}$$

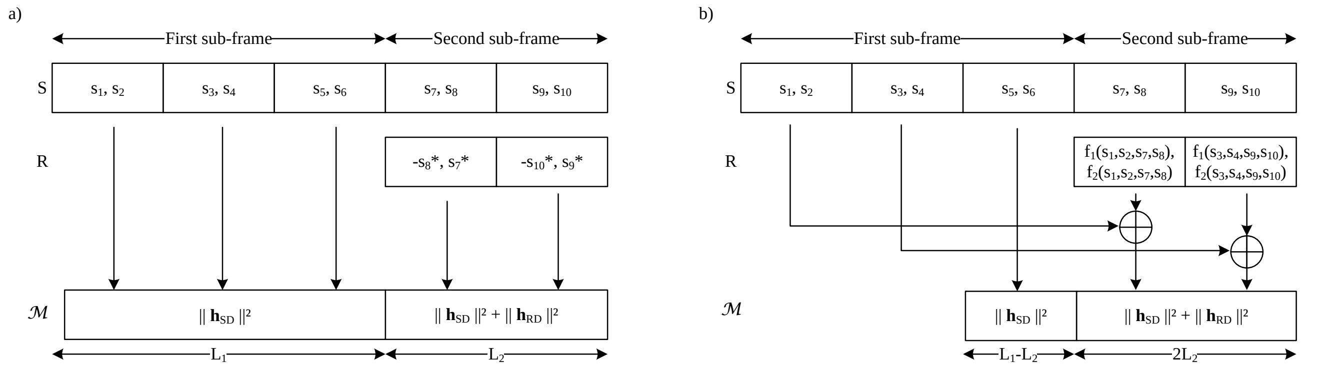

Fig. 1. Symbols transmitted by the source S and the relay R during the first and second sub-frames and resulting Matryoshka channel $\mathcal{M}$: a) using a Distributed Alamouti scheme during the second sub-frame, b) using a Patched-DSTBC during the second sub-frame and applying at the destination a combination of the symbols received during the first and second sub-frame.

where $\mathbf{H} = [\mathbf{h}_{SD} \quad \mathbf{h}_{RD}]$ is a $N_r \times 2$ matrix and $\boldsymbol{\eta}_2$ a white complex gaussian noise $N_r \times 2$ matrix with zero mean and variance $2N_0$ entries.

The destination D then combines $\mathbf{Y}_1$ and $\mathbf{Y}_2$ into $\mathbf{Y}$:

$$\mathbf{Y} = \mathbf{Y}_1\mathbf{A} + \mathbf{Y}_2\mathbf{B} \tag{3}$$

where $\mathbf{A}$ and $\mathbf{B}$ are two matrices such that

$$\mathbf{Y} = \mathbf{H}\mathbf{X} + \boldsymbol{\eta} \tag{4}$$

where $\mathbf{X}$ is a codeword of a space-time code and $\boldsymbol{\eta}$ is a white complex gaussian noise matrix with zero mean and variance $2N_0$ entries. The white gaussian noise condition is obtained by choosing $\mathbf{A}^\dagger\mathbf{A} + \mathbf{B}^\dagger\mathbf{B} = \mathbf{I}$, where $\dagger$ denotes the transpose conjugate operator.

We can remark that we consider cases where the source always transmit unmodified symbols during the second sub-frame. The practical advantage of this choice is that there is no need of signalling from the relay to the source. Furthermore, the so-called relay-unaware source doesn't even have to know the existence of the relay, which allows for including relays into systems not designed for them. The assumption of relay-unaware source is used in all the following results, which implies that the source does not adapt its transmission for the second sub-frame.

It has to be noticed that the relay transmits signals $f_1(s_1, s_2, s_3, s_4)$ or $f_2(s_1, s_2, s_3, s_4)$ being from a different codebook of the source one's, as stated by [1]. The constellation built from $f_1(s_1, s_2, s_3, s_4)$ or $f_2(s_1, s_2, s_3, s_4)$ does not necessary meet the best shaping designs. However, we consider that a power control is applied to the relay such that $SNR_{SD} = SNR_{RD}$. In practice, we often consider that the path loss from the source to the destination is much lower than the one from the relay to the destination, which gives a sufficient margin for achieving the equals SNR condition while using a bad shaped (e.g., high peak to average power ratio) constellation at the relay. Thus, the Patched-DSTBCs do not show, to our knowledge, practical issues.

Three practical examples of Patched-DSTBC are presented: the Patched-Alamouti Code, the Patched-Golden Code and the Patched-Silver Code.

A. Patched Alamouti

We assume that during the second sub-frame, the relay transmits a combination of s_1 (or s_2) from a first (or second) time-slot of the first sub-frame and s_3 (or s_4) from a first (or second) time-slot of the second sub-frame as follows [9]:

$$f_1(s_1, s_2, s_3, s_4) = (\alpha s_2 + s_4)^* \tag{5}$$
$$f_2(s_1, s_2, s_3, s_4) = -(\alpha s_1 + s_3)^* \tag{6}$$

with α a scaling factor and * the complex conjugaison operator. The destination combines $\mathbf{Y}_1$ and $\mathbf{Y}_2$ as follows:

$$\mathbf{Y} = \frac{(\alpha\mathbf{Y}_1 + \mathbf{Y}_2)}{\sqrt{1+\alpha^2}} \tag{7}$$
$$= \mathbf{H}\begin{bmatrix} \frac{s_3+\alpha s_1}{\sqrt{1+\alpha^2}} & \frac{\alpha s_2+s_4}{\sqrt{1+\alpha^2}} \\ \frac{(\alpha s_2+s_4)^*}{\sqrt{1+\alpha^2}} & \frac{-(s_3+\alpha s_1)^*}{\sqrt{1+\alpha^2}} \end{bmatrix} + \boldsymbol{\eta} \tag{8}$$

The parameter α defines an hyper-constellation $s' + \alpha s$. For example, if the symbols s and s' belong to a QPSK modulation, choosing $\alpha = 0.5$ makes $\frac{s'+\alpha s}{\sqrt{1+\alpha^2}}$ belong to a normalized 16-QAM modulation. Thus, after combination of received signals from the two sub-frames, the receiver can decode the combined signal as for a 2xN_r MIMO channel with an Alamouti scheme and 16-QAM input, which makes the scheme implementation-attractive. Unfortunately, the increased modulation cardinality induce a loss in coding gain with respect to a DA-DDF scheme achieving full diversity.

B. Patched Golden Code

The destination can realize a codeword of the Golden Code [10] during the second sub-frame of the codeword if the relay transmits

$$f_1(s_1, s_2, s_3, s_4) = i\frac{\overline{\phi}}{\phi}(s_4 + \overline{\alpha}s_2) \tag{9}$$

$$f_2(s_1, s_2, s_3, s_4) = \frac{\overline{\phi}}{\phi}(s_3 + \overline{\alpha}s_1) \tag{10}$$

with $\alpha = (1+\sqrt{5})/2$ and $\phi = 1 + i - i\alpha$, $\overline{\alpha} = (1-\sqrt{5})/2$ and $\overline{\phi} = 1 + i - i\overline{\alpha}$. The destination combines $\mathbf{Y}_1$ and $\mathbf{Y}_2$ as follows:

$$\mathbf{Y} = \frac{\phi(\alpha\mathbf{Y}_1 + \mathbf{Y}_2)}{\sqrt{|\phi^2(1+\alpha^2)|}} \tag{11}$$

$$= \frac{\sqrt{5}}{\sqrt{|\phi^2(1+\alpha^2)|}}\mathbf{H}\mathbf{X} + \boldsymbol{\eta} \tag{12}$$

with $\mathbf{X}$ being a codeword of the Golden Code:

$$\mathbf{X} = \frac{1}{\sqrt{5}}\begin{bmatrix} \phi(s_3 + \alpha s_1) & \phi(s_4 + \alpha s_2) \\ i\overline{\phi}(s_4 + \overline{\alpha} s_2) & \overline{\phi}(s_3 + \overline{\alpha} s_1) \end{bmatrix} \tag{13}$$

The scaling factor streches the golden code constellation by a factor $1/\sqrt{2}$ which corresponds to degradation of 1.5dB on the signal-to noise ratio. In other words, the coding gain brought by the golden code structure must be higher than the loss involved by the noise normalization. This will be illustrated in section V.

C. Patched Silver Code

The destination can realize a codeword of the Silver Code [11], [12] during the second sub-frame of the codeword if the relay transmits

$$f_1(s_1, s_2, s_3, s_4) = -s_4^* - \frac{(1-2i)s_1^* + (1+i)s_2^*}{\sqrt{7}} \tag{14}$$

$$f_2(s_1, s_2, s_3, s_4) = s_3^* + \frac{(-1+i)s_1^* + (1+2i)s_2^*}{\sqrt{7}} \tag{15}$$

and the destination combines $\mathbf{Y}_1$ and $\mathbf{Y}_2$ as follows:

$$\mathbf{Y} = \mathbf{Y}_1 \frac{1}{\sqrt{7}}\begin{bmatrix} 1+i & -1+2i \\ -(1+2i) & (-1+i) \end{bmatrix} + \mathbf{Y}_2 \tag{16}$$

Noting that the matrix multiplying $\mathbf{Y}_1$ is orthogonal, the resulting noise is thus uncorrelated and only a scaling factor $1/\sqrt{2}$ is needed to generate a complex gaussian noise of zero mean and variance $2N_0$. Then,

$$\mathbf{Y} = \frac{1}{\sqrt{2}}\mathbf{H}\mathbf{X} + \boldsymbol{\eta} \tag{17}$$

with $\mathbf{X}$ being a codeword of the Silver Code:

$$\mathbf{X} = \begin{bmatrix} s_3 + \frac{(1+i)s_1 - (1+2i)s_2}{\sqrt{7}} & s_4 + \frac{(-1+2i)s_1 + (-1+i)s_2}{\sqrt{7}} \\ -s_4^* + \frac{(-1+2i)s_1^* - (1+i)s_2^*}{\sqrt{7}} & s_3^* + \frac{(-1+i)s_1^* - (1+2i)s_2^*}{\sqrt{7}} \end{bmatrix} \tag{18}$$

The Patched Golden Code and the Patched Silver Code suffer from the same scaling factor of $1/\sqrt{2}$, consequently their performance should be nearly the same as observed for classical MIMO systems. Performance of these three realizations of Patched-DSTBC are studied in the following section.

V. Outage Analysis

The achievable diversity of the protocol depends on the segmentation of the codeword and on the dynamic correct decoding time at the relay. The probability of correct decoding after receiving M sub-frames is noted $P_R(M)$. Indeed, the sooner the relay correctly decodes the information word, the better the chance to achieve full diversity. Consequently, the frame error probability $p_e(M)$ exhibits different diversity order and the outage probability of the transmission is

$$P_e = \sum_{M=1}^{N_b} P_R(M) p_e(M) \tag{19}$$

For all the following results, a codeword is segmented into 2 sub-frames, the first one being twice longer than the second one, i.e. $L_1 = 2L_2$. Such a codeword thus represents a simplified DDF protocol. The source and the relay transmit using a QPSK modulation.

Note that the performance of the Silver and Golden Code being similar for classical MIMO systems, the performance of the Patched Golden Code and Patched Silver Code are similar. We will omit the Patched-Golden code in the following, as the Patched-Silver code can be decoded with less complexity.

Fig.2 illustrates the outage probability when the relay correctly decodes the message after the first sub-frame, using an error correcting code rate $R_c = 0.46$ for the two phases transmission. The destination has one receive antenna. For this coding rate, and as explained in section III, we observe that our proposed Patched-DSTBC achieves a diversity order of 2 but not the DA-DDF scheme. The Patched-DSTBC allows for guarantying full diversity while suffering from a coding gain degradation. Thus, the scenario with a minimal use of Patched-STBC exhibits a better coding gain.

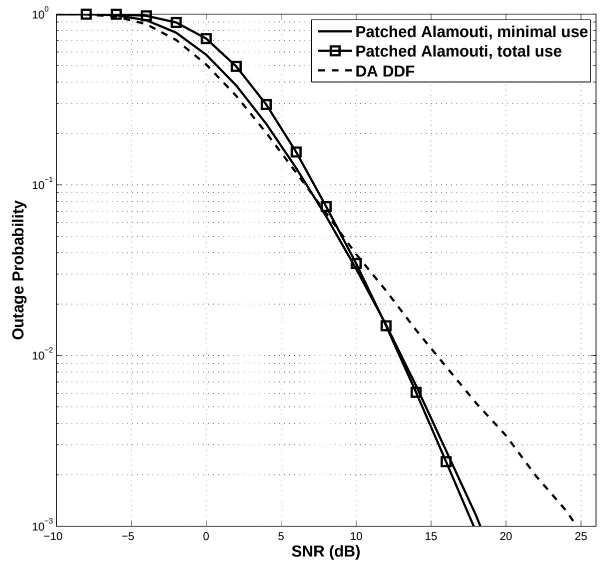

Fig. 2. Outage probability of the Patched Alamouti for total and minimal use compared to the DA DDF for two sub-frames of length $L_1 = 2L_2$, for a coding rate $R_c = 0.46$, assuming that the relay correctly decodes the message after the first sub-frame.

Fig.3 shows the outage probability of the DA-DDF, the Patched Silver Code and Patched Alamouti with minimal use, for a coding rate $R_c = 0.46$. We assume that the relay correctly decodes the message after the first sub-frame and the destination carries two reception antennas. We observe that the Patched Silver Code exhibits better performance than the

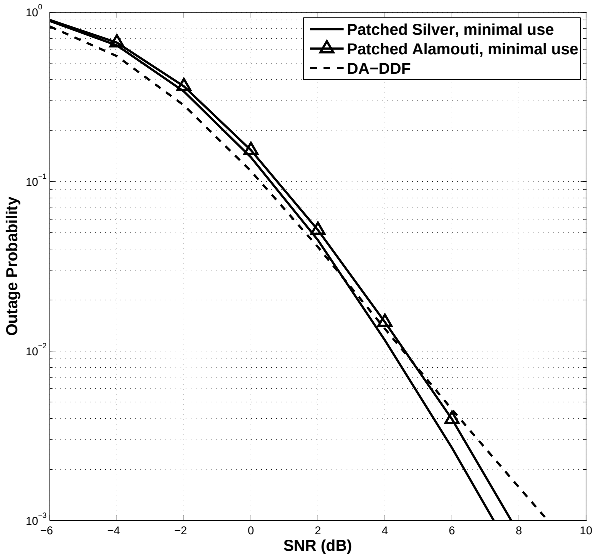

Fig. 3. Outage probability of the Patched Silver Code and Patched Alamouti with minimal use, and the DA-DDF for two sub-frames of length $L_1 = 2L_2$, for a coding rate $R_c = 0.46$, assuming that the relay correctly decodes the message after the first sub-frame.

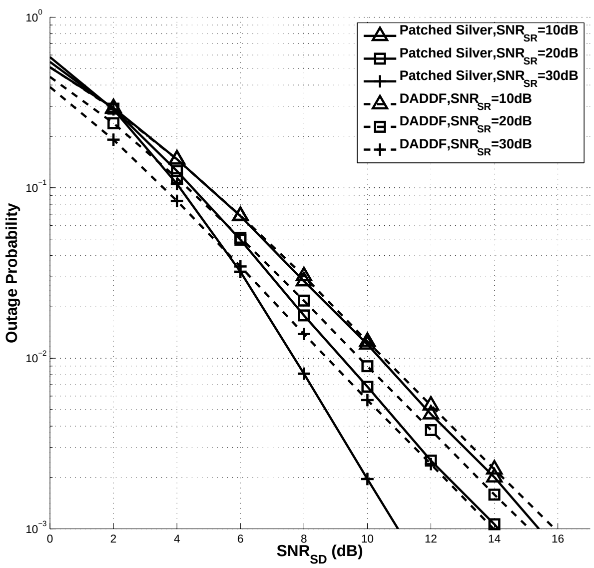

Fig. 4. Outage probability of the Patched Silver Code and the DA-DDF for two sub-frames of length $L_1 = 2L_2$, for a coding rate $R_c = 2/3$ according to different SNR_{SR}: 10dB, 20dB, 30dB.

Patched Alamouti, and allows for achieving the full-diversity order 4, contrary to the DA-DDF.

Fig.4 illustrates the outage probability of the Patched Silver Code and the DA-DDF taking into account the probability of correct decoding at the relay after receiving the first sub-frame according to different SNR_{SR}, for two reception antenna at the destination. Note that for this particular coding rate $R_c = 2/3$, the performance of the two scenarii of Patched DSTBC are the same. We observe that the Patched Silver Code achieves better performance than the DA-DDF. Moreover, the better the source-relay link is, the higher the gap. Indeed, the better the source-relay link is, the more often the relay correctly decodes the message after receving the first sub-frame, and the higher the performance is improved via the diversity order $2N_r$ brought by the Patched-STBC schemes. As a remark, the overall outage probability exhibits a diversity order N_r as the probability of correct decoding at the relay is never null.

VI. Conclusion

In this paper, we have introduced the Patched-STBC schemes that allow for observing full diversity when the classical DDF scheme using DSTBC does not. We have shown that the relay can transmit combinations of symbols from phase one and two during a portion of time of phase two. By combining the received symbols of phase one and two, the destination re-create the hidden space-time block code structure, which changes the nature of the block fading channel seen by the decoder and allows for guarantying full diversity. Finally, by using the minimal amount of Patched-STBCs allowing for guarantying full diversity, the coding gain is optimized and our scheme outperforms the known DDF schemes in all cases.

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