Signal predistortion scheme based on the contraction mapping theorem

Nicolas Alibert[1,2], Karine Amis[1], Charlotte Langlais[1], Damien Castelain[2]

[1]IMT Atlantique, Lab-STICC, Univ. Bretagne Loire, 29238 Brest, France,

[2]Wireless Communications Systems Department, Mitsubishi Electric R&D Centre Europe,
1 allée de beaulieu, CS 10806, 35708 Rennes Cedex 7, France

{n.alibert,d.castelain}@fr.merce.mee.com
{karine.amis,charlotte.langlais}@imt-atlantique.fr

Résumé – L'augmentation des débits de communications par satellite impose d'utiliser des constellations à plusieurs niveaux d'énergie, comme la 32-APSK, pour améliorer l'efficacité spectrale. Cependant, ces constellations sont plus sensibles à l'interférence entre-symboles non-linéaire, générée par l'amplificateur et les filtres embarqués, interférence qui peut être compensée à l'émission par un schéma de prédistorsion. Après modélisation du canal satellitaire, nous réécrivons le problème de prédistorsion comme un problème de recherche du point fixe. Puis, nous proposons un schéma de prédistorsion du signal basé sur le théorème du point fixe.

Abstract – The need for higher satellite throughtput leads to use multiple-ring constellations, such as the 32-APSK, in order to increase the spectral efficiency. However, these constellations are more sensitive to the non-linear intersymbol interference, generated by the HPA and on-board filters, which can be dealt with by a predistortion scheme at the transmitter. After modelling the satellite channel, we derive a fixed-point formulation of the predistortion problem. Then, we propose a signal predistortion scheme based on the contraction mapping theorem that compensates for the non-linear interference.

1 Introduction

The need for higher satellite throughput and for reduced cost per bit can be solved by using spectrally-efficient modulation schemes, such as 32-APSK, and by making better use of on-board power amplifier. The problem with multiple-ring modulation is that they are more sensitive to non-linear interference. The amplifier which introduces non-linearity also induces spectral regrowth that results in adjacent carriers interference. Compensation of non-linearity is then one key to achieve higher throughput.

There are multiple ways to mitigate the non-linear interference, either at the transmitter with predistortion [1]–[6] or at the receiver with equalization [7]. Among the predistortion techniques, we can find in the literature two kinds of predistortion: data predistortion [1]–[4] and signal predistortion [5], [6]. In this paper, we focus on signal predistortion.

To the best of our knowledge, the best performance is achieved by the signal predistortion scheme proposed in [6] and used as the reference scheme. It is based on successive approximations. One drawback of that method is that a decreasing step-size has to be empirically optimized.

In this paper, we propose a signal predistortion based on a fixed-point problem formulation. In [3], a fixed-point predistortion scheme has been proposed for a non-linear channel only composed of the non-linear amplifier, which is a memoryless non-linear channel. In this article, we extend the method to a Volterra filter that is a model for non-linear channel with memory. Contrary to [8], our formulation, doesn't require the inversion of the linear part of the channel, which is computationally expensive. Thanks to the contraction mapping theorem, the solution of the fixed-point problem is given by a recursive formula. Based on that recursion, we construct a signal predistortion scheme which slightly outperforms the reference scheme of [6]. However, the proposed scheme doesn't require the empirical optimization of the step-size and the convergence is shown by the contraction mapping theorem.

The article is organized as follows. Section 2 describes the system model and the hypotheses. In section 3, the state-of-the-art method introduced by [6] is described. The section 4 derives the fixed-point problem, then based on the recursion found, the signal predistortion scheme is proposed in section 5. A comparison of the reference [6] and the proposed one is made in section 6. Finally, the paper is concluded in section 7.

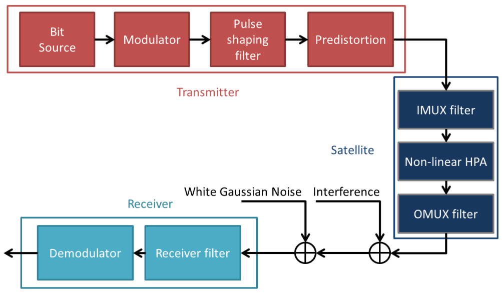

Figure 1 – System model

2 System model and hypotheses

Figure 1 shows the block diagram of the satellite system. We focus on a single carrier per transponder scheme. In our application, both transmit and receiver filters are square-root raised cosine (SRRC) filters. The uplink channel is assumed noise-free. The satellite transponder consists of the input multiplexer (IMUX), the high power amplifier (HPA) and the output multiplexer (OMUX). The HPA is a memoryless non-linear component. Therefore, the whole system behaves as a nonlinear system with memory [9]. The IMUX selects the carrier under interest, while, the OMUX filter limits the spectral regrowth due to the distortion and reduces the spillage over adjacent carriers. On the downlink, we consider interference from adjacent carriers (ACI).

We assume that the overall channel can be modelled as a Volterra filter as in [10].

3 Successive approximation method [6]

In [6], the author proposed different schemes based on a stochastic approximation type iteration. Among the three schemes proposed in [6], we consider the scheme that minimizes the in-band distortion as a reference for ours.

The scheme is based on the stochastic approximation, which is usually used for finding zeros of an unknown function when only noisy measurements are available. The predistorted signal is found iteratively. The output of the k-th stage is given by:

$$\bar{x}^{(k+1)}(t) = \bar{x}^{(k)}(t) + \mu^{(k)} e^{(k)}(t), \tag{1}$$

where $\bar{x}^{(k)}$ is the predistorted signal, $e^{(k)}$ is an error signal and $\mu^{(k)}$ is the k-th step-size.

The method is initialized by the undistorted signal,

$$\bar{x}^{(0)}(t) = x(t). \tag{2}$$

The sequence of step-sizes $\{\mu^{(k)}\}$ are chosen accordingly to the following rules:

$$\mu^{(k)} > 0, \mu^{(k)} \to 0, \sum_{k=0}^{\infty} \mu^{(k)} = \infty, \sum_{k=0}^{\infty} (\mu^{(k)})^2 < \infty. \tag{3}$$

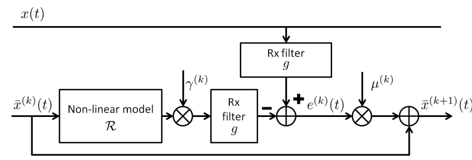

Figure 2 – Reference scheme [6]

A general expression that satisfies (3) is given in [11]:

$$\mu^{(k)} = \mu^{(0)} \frac{(\frac{b}{k+1} + a)}{(a + \frac{b}{k+1} + (k+1)^c - 1)}, \tag{4}$$

where the quadruplet $(\mu^{(0)}, a, b, c)$ has to be optimized.

At each step k the error is computed as follows:

$$e^{(k)}(t) = g(t) \star (x(t) - \gamma^{(k)} \mathcal{R}\{\bar{x}^{(k)}(t)\}), \tag{5}$$

where $\star$ denotes the convolution, and

$$\gamma^{(k)} = \frac{\int \mathcal{R}\{\bar{x}^{(k)}(t)\}^* x(t) \mathrm{d}t}{\int |\mathcal{R}\{\bar{x}^{(k)}(t)\}|^2 \mathrm{d}t}, \tag{6}$$

$\gamma^{(k)}$ is a phase and amplitude correction, $\mathcal{R}\{.\}$ is the Volterra filter which models the IMUX, HPA and the OMUX, $g(t)$ is the receiver filter and the superscript $*$ is the complex conjugate.

Figure 2 illustrates the scheme proposed in [6].

4 Proposed data predistortion based on fixed-point approach

We assume that the channel can be mathematically described as a Volterra filter, denoted by $\mathcal{F}$. This assumption is not restrictive as long as the non-linear HPA can be decomposed as a Taylor series [10].

The problem of the predistortion can be formulated as:

$$\mathcal{F}\{\bar{\boldsymbol{d}}\} = \boldsymbol{d}. \tag{7}$$

where $\mathcal{F}$ is the nonlinear operator, $\boldsymbol{d}$ is the sequence of symbols to be transmitted and $\bar{\boldsymbol{d}}$ is the sequence of predistorted symbols to be found. Equation (7) can be written as a fixed-point problem formulation:

$$(\mathcal{I} - \mathcal{F})\{\bar{\boldsymbol{d}}\} + \boldsymbol{d} = \bar{\boldsymbol{d}}, \tag{8}$$

where $\mathcal{I}$ is the identity operator.

If the operator $\mathcal{T}\{.\} = (\mathcal{I} - \mathcal{F})\{.\} + \boldsymbol{d}$ is a contraction mapping then according to the contraction mapping theorem, the solution is unique and can be reached iteratively by [3], [8]:

$$\bar{\boldsymbol{d}}^{(k+1)} = \mathcal{T}\{\bar{\boldsymbol{d}}^{(k)}\}. \tag{9}$$

The condition for the operator $\mathcal{T}$ to be a contraction can be found in [8]. The convergence is validated numerically in section 6. Even if the hypothesis is not satisfied, a few iterations might still improve the performance [8]. Otherwise, as in [3], a constant step-size can be introduced to ensure that $\mathcal{T}$ is a contraction mapping.

The recursion can be written as follows:

$$\bar{\boldsymbol{d}}^{(k+1)} = \bar{\boldsymbol{d}}^{(k)} + (\boldsymbol{d} - \mathcal{F}\{\bar{\boldsymbol{d}}^{(k)}\}). \tag{10}$$

Figure 3 shows the stage for symbol predistortion.

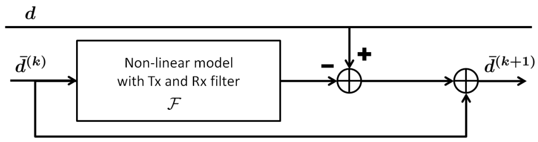

Figure 3 – Data predistortion structure

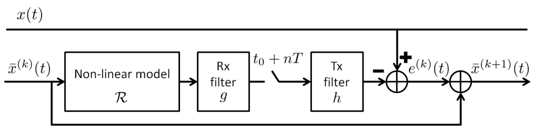

Figure 4 – Proposed scheme

5 Proposed signal predistortion

In this section, we derive a signal predistorsion scheme from the contraction mapping theorem. We first establish the connection between the data-based model and the signal-based model.

By denoting by $h(t)$ the transmit low-pass filter, the modulated signal $x(t)$ reads

$$x(t) = \sum_n d_n h(t - nT), \tag{11}$$

where T is the symbol duration.

$x(t)$ is the input of the satellite transponder that can be modelled by the non-linear operator $\mathcal{R}$. The relation between $\mathcal{F}$ and $\mathcal{R}$ is thus given by:

$$\mathcal{F}\{\boldsymbol{d}\} = \left\{ \Big[g(t) \star \mathcal{R}\{x(t)\} \Big]_{t=t_0+nT} \right\}, \tag{12}$$

where $g(t)$ is the receiver filter and t_0 is the optimal sampling time instant (minimum distortion).

Using (10) and exploiting the fact that h and g are SRRC filters, the equivalence between the signal and the data predistortion reads:

$$\begin{aligned}\bar{x}^{(k+1)}(t) &= \bar{x}^{(k)}(t) + x(t) \\ &\quad - \sum_n \Big[g(t) \star \mathcal{R}\{\bar{x}^{(k)}(t)\} \Big]_{t=t_0+nT} h(t - nT),\end{aligned} \tag{13}$$

yielding to the error signal:

$$e^{(k)}(t) = x(t) - \sum_n \Big[g(t) \star \mathcal{R}\{\bar{x}^{(k)}\} \Big]_{t=t_0+nT} h(t-nT). \tag{14}$$

The resulting proposed signal predistortion scheme is illustrated in figure 4.

6 Performance Evaluation

In the following, we compare the performance of the signal predistortion based on the contraction mapping theorem with the successive approximation method proposed in [6] through Monte Carlo simulations.

The characteristics of the IMUX, OMUX filters and the traveling-wave tube amplifier (TWTA) are the one proposed in the DVB-S2 [12]. The bit stream is mapped onto an 32-APSK constellation defined in [12]. The output symbols are sent at 38 MBd. SRRC filters are used with a 5% roll-off. An over-sampling factor of 8 is used. Two adjacent carriers are located at 40 MHz on either sides of the carrier of interest. The interferers are delayed and frequency-shifted versions of the OMUX output signal.

For the predistortion, we assume ideal knowledge of the non-linear channel.

In the simulations, the reference scheme [6] is applied with a genie-aided amplitude and phase correction at the receiver to remove the wrapping and we consider two step-size sequences defined by (4) with $\mu^{(0)} = 1$. $(a, b, c) = (1, 0, 0)$ (yielding $\mu^{(k)} = 1$) for the first one and $(a, b, c) = (10, 0, 0.6)$ for the second one.

6.1 Figure of merit

6.1.1 Total degradation

A commonly used performance metric in order to evaluate the quality of the compensation of the nonlinearity and the use of the amplifier is the total degradation. For the chosen bit error rate (BER), the total degradation is expressed as follows:

$$TD = OBO + \left.\frac{E_b}{N_0}\right|_{NL} - \left.\frac{E_b}{N_0}\right|_{AWGN} \quad \text{[dB]}. \tag{15}$$

The output back-off (OBO) is the difference between the amplifier's maximum output power and the mean output power. We measure the OBO at the output of the receiver filter. The $\left.\frac{E_b}{N_0}\right|_{NL}$ (respectively $\left.\frac{E_b}{N_0}\right|_{AWGN}$) is average bit energy over noise ratio required to achieve the targeted BER in the non-linear (resp. in the additive white gaussian noise) channel.

6.1.2 NMSE

The NMSE measures the in-band distortion at the receiver and can be expressed as:

$$NMSE = 10 \log \left(\mathrm{E} \left(\frac{\sum_n |\hat{d}_n - d_n|^2}{\sum_n |d_n|^2} \right) \right) \quad \text{[dB]}, \tag{16}$$

where E is the mathematical expectation, d_n is the n-th transmitted symbol and $\hat{d}_n$ is the n-th sampler output.

6.2 Results

The Total Degradation and the NMSE are compared in Fig. 5 and Fig. 6 respectively.

Concerning the total degradation, the performance from the method proposed in [6] depends on the chosen step-size parameters (eq. (4)) and requires more iterations (20 ite.) than ours (5 ite.).

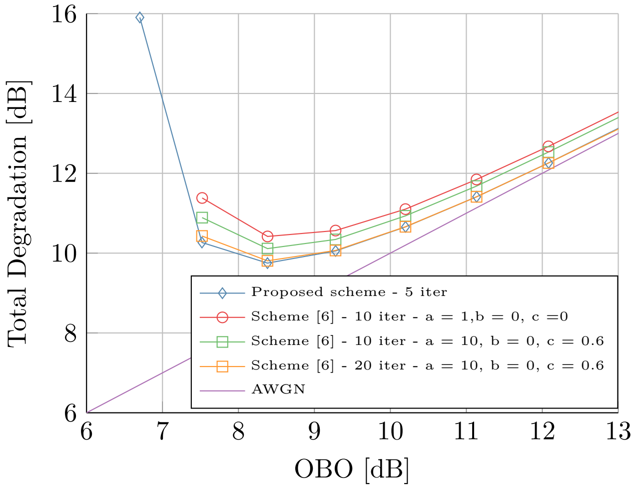

Figure 5 – Total degradation for a target BER of 10^{-4}.

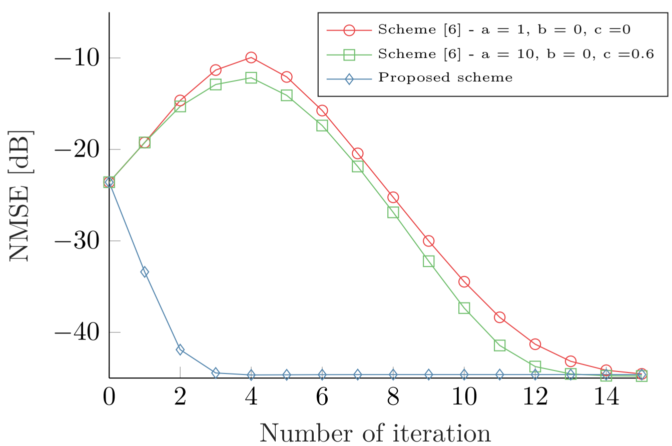

Figure 6 – NMSE at the ouput of the matched filter for an IBO of 14 dB versus the number of stages

As for the NMSE, the proposed algorithm requires 5 iterations to converge when the reference scheme needs more than 10 iterations to achieve the same NMSE.

7 Conclusion

In this article, we have derived a symbol-based fixed-point approach of the predistortion problem. This formulation is performed without the need of computing the inverse of the linear part as in [8]. Based on that, a signal predistortion structure has been proposed. The proposed scheme is obtained thanks to the contraction mapping theorem, which ensures the convergence. Moreover, compared to the reference scheme [6] we show that our method achieves good performance without the need of empirically optimizing step-size parameters.

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