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Hybrid Precoding for Wideband Multi-user MIMO Millimeter Wave System

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***Abstract*—Hybrid precoding for a wideband multi-user multiple input multiple output (MIMO) millimeter wave (mmWave) system is more challenging than the narrowband design since the radio frequency (RF) precoder and combiner are frequency non-selective and they should be optimized with respect to all scheduled users on all allocated subcarriers. In this work, we adopt the separate stage hybrid precoding strategy and propose two algorithms with good performance and complexity trade-off. Based on the proposed strategies of minimal equivalent channel gain maximization or average wideband sum rate maximization, the non-convex RF beamforming can be solved first by a semi-definite relaxation and codebook projection. In the second stage, conventional sum rate maximization digital precoder/combiner can be obtained based on smaller dimension equivalent channel feedback. Simulations shows that the proposed algorithms outperform the state of art algorithms in multiple receive antennas setting.**

I. Introduction

Communication over the mmWave band is a key enabler for high throughput transmission in 5G cellular system. However, the precoder and combiner design for wideband multi-user massive MIMO mmWave System is non trivial due to the hybrid analog/digital structure. Apart from the common difficulties such as analog-digital precoding split and discrete analog beam choice [1], the fact that the analog precoding parts are shared among all scheduled users on all operating subcarriers adds further constraints for the hybrid design.

Hybrid precoding has been extensively studied in many prior works [2]–[17]. They can be cataloged with different labels: single user (SU) design, multi-user (MU) design, narrowband design and wideband design. Different labels can be combined together, such as SU-narrowband design [2]–[5], SU-wideband design [6]–[8], MU-narrowband design [9]–[12] and MU-wideband design [8], [13]–[17].

Generally speaking, two approaches have been used in prior art for wideband MU-MIMO hybrid precoding: (i) factorization based design (ii) separate/iterative two stage design. Factorization based design [14]–[16] starts from the optimal full digital precoder and tries to find the RF and baseband precoder sufficiently close to the full digital precoder. Separate/iterative two stage design adopts different objective functions on each stage. For example, [14] applies signal noise ratio (SNR) maximization for receive signal as RF precoding criterion and wideband sum rate maximization as digital precoding criterion, an alternating optimization is performed between the two stages to improve the performance. In [17], minimization of a signal to leakage plus noise ratio (SLNR) metric is used. In [8], the RF precoding criterion is to maximize the upper bound of an equivalent channel capacity. It's hard to draw a simple conclusion on which method is better. Factorization based methods can achieve a sum rate performance which is close to the full digital design. However, these designs require heavy channel state information (CSI) overhead exchange. Separate/iterative two stage design could have simpler (even closed-form in some cases) precoding design and limited CSI overhead for certain objective functions. However, there is a non-vanishing performance gap compared with full digital precoding and the convergence is not guaranteed for certain iterative designs due to the fact that different design criteria are used at each stage. In this work, we focus on the two stage design and try to propose criteria which have good complexity-performance trade-off.

In the state of the art separate/iterative two stage design, many works [8], [17] only focus on users with single antenna. Another important assumption is whether different user multiplexing can be used on different subcarriers. It is reported in [8] that it's often more spectrum efficient to allocate the same user onto the entire band since the channels of different subcarriers are highly correlated due to the sparse nature of the mmWave channel. Therefore, many works (e.g., [8], [14]–[17]) adopt this assumption and try to optimize the frequency flat analog precoder/combiner which beamforms properly all frequency selective channels on each subcarrier. However, this assumption is not always valid. Firstly, the system can suffer from the effect of beam squint [13] when large bandwidth signals are considered. In this case, the column space spanned by the steering directions on different subcarriers are not perfectly aligned for the concerned user. Therefore, allocating the same user over all subcarriers is not advantageous. Secondly, the scheduler criterion and users' traffic model could have an impact on the user multiplexing on different subcarriers. For example, if some users have short packets or fairness is emphasized among users, different user multiplexing on distinct subcarriers can be more suitable. To cope with the beam squint effect, an efficient joint scheduling and hybrid precoding design which allocates successively an additional stream, eliminates the interference between current and previously allocated streams and selects best users on each subcarrier is proposed in [13]. However, the proposed algorithm can be hardly generalized to satisfy certain scheduler design criteria such as user fairness. In this work, we propose some new RF precoding criteria under arbitrary users scheduling over multiple subcarriers, each user can have arbitrary antennas/RF chains/streams configuration. The propose RF precoding criteria also have good complexity-performance trade-off.

II. System and Channel Model

A. System Model

We consider a wideband multi-user mmWave communication system which can be decomposed into L equivalent narrowband channel. A transmitter (TX) equipped with N_t transmit antennas will jointly serve K receivers (RX) on each subcarrier. Each scheduled RX is equipped with N_r receive antennas and receives N_s data streams from the TX. The total number of streams transmitted by the TX is KN_s. We assume that the TX has L_t transmit RF chains and each RX has L_r receive RF chains. In order to fulfill the aforementioned transmission scenario, the constraints $N_t \geq L_t \geq KN_s$ and $N_r \geq L_r \geq N_s$ are assumed. At the TX, data streams will be processed by a frequency selective base band precoder $\mathbf{F}_{\rm BB}[\ell] \in \mathbb{C}^{L_t \times KN_s}$ followed by a $N_t \times L_t$ dimension frequency flat RF precoder $\mathbf{F}_{\rm RF}$. For the kth RX scheduled on the ℓth subcarrier denoted as RX $\pi(\ell,k)$, the receiving data streams pass through a $N_r \times L_r$ dimension frequency flat RF combiner $\mathbf{W}_{{\rm RF},\pi(\ell,k)}$ followed by a frequency selective base band combiner $\mathbf{W}_{{\rm BB},\pi(\ell,k)}[\ell] \in \mathbb{C}^{L_r \times N_s}$.

Therefore, the the narrow band transmission for the signal received at RX $\pi(\ell,k)$ is

$$\begin{aligned}\boldsymbol{y}_{\pi(\ell,k)}[\ell] = &\mathbf{W}^{\rm H}_{{\rm BB},\pi(\ell,k)}[\ell]\mathbf{W}^{\rm H}_{{\rm RF},\pi(\ell,k)}\mathbf{H}_{\pi(\ell,k)}[\ell]\mathbf{F}_{\rm RF}\mathbf{F}_{\rm BB}[\ell]\boldsymbol{s}[\ell]\\ &+ \mathbf{W}^{\rm H}_{{\rm BB},\pi(\ell,k)}[\ell]\mathbf{W}^{\rm H}_{{\rm RF},\pi(\ell,k)}\boldsymbol{n}_{\pi(\ell,k)}[\ell],\end{aligned}$$

where $\boldsymbol{s}[\ell] = \left[\boldsymbol{s}^{\rm H}_{\pi(\ell,1)}[\ell], \ldots, \boldsymbol{s}^{\rm H}_{\pi(\ell,K)}[\ell]\right]^{\rm H} \in \mathbb{C}^{KN_s \times 1}$ is the concatenation of the data symbols for all the K co-scheduled RXs on subcarrier ℓ. The power of data symbol vector satisfies $\mathbb{E}\left(\boldsymbol{s}[\ell]\boldsymbol{s}^{\rm H}[\ell]\right) = \mathbf{I}_{KN_S}$. Data symbol for RX $\pi(\ell,k)$ is $\boldsymbol{s}_{\pi(\ell,k)}[\ell] \in \mathbb{C}^{N_s \times 1}$. The receive signal at RX $\pi(\ell,k)$ is $\boldsymbol{y}_{\pi(\ell,k)}[\ell] \in \mathbb{C}^{N_s \times 1}$ and $\boldsymbol{n}_{\pi(\ell,k)}[\ell] \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}, \sigma^2\mathbf{I}_{N_r})$ is the Gaussian noise vector for RX $\pi(\ell,k)$. $\mathbf{H}_{\pi(\ell,k)}[\ell] \in \mathbb{C}^{N_r \times N_t}$ is the user channel for RX $\pi(\ell,k)$.

The RF precoder $\mathbf{F}_{\rm RF}$ and RF combiner $\mathbf{W}_{{\rm RF},\pi(\ell,k)}$ are assumed to be implemented by phase shifting networks [18] where each transceivers is connected to each antenna through a network of phase shifters with finite resolution. This indicates that each column of $\mathbf{F}_{\rm RF}$ and $\mathbf{W}_{{\rm RF},\pi(\ell,k)}$ has constant modulus and the angles belong to predefined finite quantization phase sets $\Phi_{\rm Prec}$ and $\Phi^{\pi(\ell,k)}_{\rm Comb}$. To show these properties, we assume the (m,n) entry of $\mathbf{F}_{\rm RF}$ and $\mathbf{W}_{{\rm RF},\pi(\ell,k)}$ satisfies

$$\begin{aligned}\left[\mathbf{F}_{\rm RF}\right]_{m,n} &= \frac{1}{\sqrt{N_t}}e^{{\rm j}\varphi^{\rm Prec}_{m,n}}, \varphi^{\rm Prec}_{m,n} \in \Phi_{\rm Prec}\\ \left[\mathbf{W}_{{\rm RF},\pi(\ell,k)}\right]_{m,n} &= \frac{1}{\sqrt{N_r}}e^{{\rm j}\varphi^{{\rm Comb},\pi(\ell,k)}_{m,n}}, \varphi^{{\rm Comb},\pi(\ell,k)}_{m,n} \in \Phi^{\pi(\ell,k)}_{\rm Comb}.\end{aligned}$$

The total power constraint is $\|\mathbf{F}_{\rm RF}\mathbf{F}_{\rm BB}[\ell]\|^2_F = P$, where P is the average total transmit power on each subcarrier.

We assume that perfect local CSIR is available at each RX $\pi(\ell,k)$ for the channel $\mathbf{H}_{\pi(\ell,k)}[\ell]$. This CSI could be obtained by a downlink channel estimation procedure using pilots. If the sparse nature of mmWave channel is further exploited, fewer pilots can be used for the channel estimation [19].

B. Channel Model

Consider the TX and RXs are equipped with uniform linear arrays (ULA). Let $\mathbf{a}_t(\phi)[\ell] \in \mathbb{C}^{N_t \times 1}$ and $\mathbf{a}_r(\phi)[\ell] \in \mathbb{C}^{N_r \times 1}$ denote the steering vectors associated to transmitter and receiver arrays, respectively. For a ULA of size N antennas, the steering vector can be written as

$$\mathbf{a}(\phi)[\ell] = \frac{1}{\sqrt{N}}[1, e^{-j2\pi\nu_\ell \sin(\phi)}, \cdots, e^{-j2\pi\nu_\ell(N-1)\sin(\phi)}]^T,$$

where $\nu_\ell = d/\lambda_\ell$, λ_ℓ is the wavelength at operating frequency ℓ, d is the antenna spacing, and ϕ is the azimuth angle.

The narrowband channel response matrix for RX i on subcarrier ℓ can be expressed as

$$\mathbf{H}_i[\ell] = \sum_{p=1}^{P}\sqrt{N_tN_r}a_{i,p}[\ell]\mathbf{a}_R(\phi_{R,i,p})[\ell]\mathbf{a}^{\rm H}_T(\phi_{T,i,p})[\ell]$$

where P denotes the number of paths, $a_{i,p}[\ell]$ is a complex amplitude, and $\phi_{R,i,p}$ and $\phi_{T,i,p}$ are azimuth angles associated with receiver and transmitter for RX i for the pth path, respectively.

III. Hybrid precoding design for Wideband MU-MIMO mmWave system

A. Sum rate maximization wideband design

Assume linear precoding and minimal mean square error (MMSE) digital receive filter is used at RX side, the average wideband sum rate performance metric for the wideband system can be denoted as

$$\begin{aligned}&\frac{1}{L}\sum_{\ell=1}^{L}\sum_{k=1}^{K}R_{\pi(\ell,k)}[\ell]\\ &= \frac{1}{L}\sum_{\ell=1}^{L}\sum_{k=1}^{K}\log\det\Big(\mathbf{I} + \mathbf{F}^{\rm H}_{{\rm BB},\pi(\ell,k)}\mathbf{H}^{\rm H}_{{\rm eq},\pi(\ell,k)}[\ell]\\ &\cdot \mathbf{R}^{-1}_{\pi(\ell,k)}[\ell]\mathbf{H}_{{\rm eq},\pi(\ell,k)}[\ell]\mathbf{F}_{{\rm BB},\pi(\ell,k)}\Big). \qquad (1)\end{aligned}$$

For scheduled RX $\pi(\ell,k)$, the effective noise covariance matrix $\mathbf{R}_{\pi(\ell,k)}$, the equivalent RF beamformed channel $\mathbf{H}_{{\rm eq},\pi(\ell,k)}[\ell]$ and the digital receive filter $\mathbf{W}_{{\rm BB},\pi(\ell,k)}[\ell]$ reads

$$\begin{aligned}\mathbf{R}_{\pi(\ell,k)} = &\sum_{i\neq\pi(\ell,k)}\mathbf{H}_{{\rm eq},\pi(\ell,k)}[\ell]\mathbf{F}_{{\rm BB},i}[\ell]\mathbf{F}^{\rm H}_{{\rm BB},i}[\ell]\mathbf{H}^{\rm H}_{{\rm eq},\pi(\ell,k)}[\ell]\\ &+ \sigma^2\mathbf{W}^{\rm H}_{{\rm RF},\pi(\ell,k)}\mathbf{W}_{{\rm RF},\pi(\ell,k)}, \qquad (2)\end{aligned}$$

$$\mathbf{H}_{{\rm eq},\pi(\ell,k)}[\ell] = \mathbf{W}^{\rm H}_{{\rm RF},\pi(\ell,k)}\mathbf{H}_{\pi(\ell,k)}[\ell]\mathbf{F}_{\rm RF}, \qquad (3)$$

$$\begin{aligned}\mathbf{W}_{{\rm BB},\pi(\ell,k)}[\ell] = &\Big(\mathbf{H}_{{\rm eq},\pi(\ell,k)}[\ell]\mathbf{F}_{{\rm BB},\pi(\ell,k)}[\ell]\mathbf{F}^{\rm H}_{{\rm BB},\pi(\ell,k)}[\ell]\mathbf{H}^{\rm H}_{{\rm eq},\pi(\ell,k)}[\ell]\\ &+\mathbf{R}_{\pi(\ell,k)}\Big)^{-1}\mathbf{H}_{{\rm eq},\pi(\ell,k)}[\ell]\mathbf{F}_{{\rm BB},\pi(\ell,k)}[\ell]. \qquad (4)\end{aligned}$$

The digital precoder $\mathbf{F}_{\mathrm{BB},i}[\ell]$ is the submatrix of $\mathbf{F}_{\mathrm{BB}}[\ell]$ which is dedicated to RXi's streams on subcarrier ℓ, $\mathbf{F}_{\mathrm{BB}}[\ell] = [\mathbf{F}_{\mathrm{BB},\pi(\ell,1)}[\ell], \cdots, \mathbf{F}_{\mathrm{BB},\pi(\ell,K)}[\ell]]$.

According to (1) and (4), It can be noticed that the optimal digital precoder and combiner that maximize the average wideband sum rate performance metric depend on the frequency selective equivalent user channel after analog beamforming. Therefore, the optimal wideband design involves intactable non-convex mixed integer joint optimization for digital and analog precoding. Besides the problem intractability, there is also the problem of heavy CSI signaling: CSI for all scheduled RXs on all subcarriers should be gathered at TX for the hybrid design. The aforementioned facts render the two stage algorithms that separate the analog precoder/combiner design from the digital precoder/combiner design. However, as is discussed in the introduction, it's non-trivial to find RF design criteria with good performance-complexity trade-off.

B. RF beamforming design: minimal equivalent channel gain maximization

In the MU-narrowband two stage design, a well-known RF beamforming design criterion is to maximize the equivalent channel gain [9], [12]. In the MU-wideband system, since the frequency non-selective RF precoder and combiner are shared between all subcarriers, a natural generalization for MU-wideband system is to maximize the minimal equivalent channel gain over all subcarriers.

Assume the transmit RF chains, receive RF chains and number of streams per RX satisfy $L_t = KL_r, L_r = N_s$, i.e., each one of the K jointly served RXs at each subcarrier will have the same number of streams as their recieve RF chains. Assume each column of the RF precoder (combiner) is selected from RF codebook $\mathcal{C}_{\mathbf{F}_{\mathrm{RF}}}$ ($\mathcal{C}_{\mathbf{W}_{\mathrm{RF}}}$), respectively. The proposed criterion reads

$$\begin{aligned}
\max_{\substack{\mathbf{F}_{\mathrm{RF},k}\\ \mathbf{W}_{\mathrm{RF},\pi(\ell,k)}}} \min_{\ell,k} \quad & \|\mathbf{W}^{\mathrm{H}}_{\mathrm{RF},\pi(\ell,k)}\mathbf{H}_{\pi(\ell,k)}[\ell]\mathbf{F}_{\mathrm{RF},k}\|_{\mathrm{F}} \\
\text{s.t.} \quad & \left[\mathbf{W}_{\mathrm{RF},\pi(\ell,k)}\right]_{(:,i)} \in \mathcal{C}_{\mathbf{W}_{\mathrm{RF}}}, \forall i = 1,\cdots,L_r \\
& \left[\mathbf{F}_{\mathrm{RF},k}\right]_{(:,i)} \in \mathcal{C}_{\mathbf{F}_{\mathrm{RF}}} \\
& \mathrm{rank}(\mathbf{F}_{\mathrm{RF},k}) = L_r \\
& \mathrm{rank}(\mathbf{W}_{\mathrm{RF},\pi(\ell,k)}) = L_r,
\end{aligned} \tag{P1}$$

where $\mathbf{F}_{\mathrm{RF},k}$ is the submatrix of $\mathbf{F}_{\mathrm{RF}}$ corresponds to the kth RX on each subcarrier, $\mathbf{F}_{\mathrm{RF}} = [\mathbf{F}_{\mathrm{RF},1}, \cdots, \mathbf{F}_{\mathrm{RF},K}]$.

To solve the discrete optimization problem (P1), we propose a sub-optimal solution which (i) relaxes the codebook constraints, and (ii) finds the closest codeword in the codebook to approximate the result in the first step. Therefore, let $\mathbf{X}_k = \mathbf{F}_{\mathrm{RF},k}\mathbf{F}^{\mathrm{H}}_{\mathrm{RF},k}, \mathbf{Y}_{\pi(\ell,k)} = \mathbf{W}_{\mathrm{RF},\pi(\ell,k)}\mathbf{W}^{\mathrm{H}}_{\mathrm{RF},\pi(\ell,k)}$, we solve first the following optimization problem:

$$\begin{aligned}
\max_{\substack{\mathbf{X}_k\\ \mathbf{Y}_{\pi(\ell,k)}}} \min_{\ell,k} \quad & \mathrm{tr}\left(\mathbf{Y}_{\pi(\ell,k)}\mathbf{H}_{\pi(\ell,k)}[\ell]\mathbf{X}_k\mathbf{H}^{\mathrm{H}}_{\pi(\ell,k)}[\ell]\right) \\
\text{s.t.} \quad & \mathrm{tr}\left(\mathbf{X}_k\right) = L_r \\
& \mathrm{tr}\left(\mathbf{Y}_{\pi(\ell,k)}\right) = L_r \\
& \mathrm{rank}(\mathbf{X}_k) = L_r \\
& \mathrm{rank}(\mathbf{Y}_{\pi(\ell,k)}) = L_r \\
& \mathbf{X}_k \succ 0, \mathbf{Y}_{\pi(\ell,k)} \succ 0,
\end{aligned} \tag{P2}$$

The discrete search space becomes continuous in (P2). It should be noticed that each RF precoder/combiner column is a unit norm DFT vector in a typical RF beamforming implementation [20]. Therefore, the norm constraints for RF precoder/combiner are preserved in aforementioned relaxation.

Solving (P2) is still non-trivial. It's a non-convex optimization due to the rank constraints. We propose to solve (P2) via an alternating maximization between the RF precoder and the RF combiner design. In each iterative step, conventional semi-definite relaxation (SDR) techniques [21] can be used. With SDR method, the rank constraints are dropped first and the optimization problem becomes convex semi-definite program. However, since we cannot always guarantee that the optimal SDR solution verifies the original rank constraint, randomization [21] or rank deduction [22] techniques should be used. The RF design based on minimal equivalent channel gain maximization is described in Algorithm 1.

C. RF beamforming design: rate lower bound maximization

Maximize the equivalent channel gain will increase the receive power level and therefore it is a valid criterion for RF beamforming. However, Since the overall design goal is to maximize the average wideband sum rate defined in (1), we propose the following RF design criterion which maximize the wideband sum rate lower bound:

$$\begin{aligned}
\max_{\substack{\mathbf{F}_{\mathrm{RF},k}\\ \mathbf{W}_{\mathrm{RF},\pi(\ell,k)}}} \quad & \tfrac{1}{L}\textstyle\sum_{\ell=1}^{L}\sum_{k=1}^{K}\log\det\Big(\tfrac{P}{Lr}\mathbf{W}^{\mathrm{H}}_{\mathrm{RF},\pi(\ell,k)}\mathbf{H}_{\pi(\ell,k)}[\ell] \\
& \cdot \mathbf{F}_{\mathrm{RF},k}\mathbf{F}^{\mathrm{H}}_{\mathrm{RF},k}\mathbf{H}^{\mathrm{H}}_{\pi(\ell,k)}[\ell]\mathbf{W}_{\mathrm{RF},\pi(\ell,k)} + \mathbf{I}\Big) \\
\text{s.t.} \quad & \left[\mathbf{W}_{\mathrm{RF},\pi(\ell,k)}\right]_{(:,i)} \in \mathcal{C}_{\mathbf{W}_{\mathrm{RF}}}, \forall i = 1,\cdots,L_r \\
& \left[\mathbf{F}_{\mathrm{RF},k}\right]_{(:,i)} \in \mathcal{C}_{\mathbf{F}_{\mathrm{RF}}} \\
& \mathrm{rank}(\mathbf{F}_{\mathrm{RF},k}) = L_r \\
& \mathrm{rank}(\mathbf{W}_{\mathrm{RF},\pi(\ell,k)}) = L_r,
\end{aligned} \tag{P3}$$

This lower bound is a generalization for the MU-narrowband RF design criterion we proposed in [23]. For a fixed RF stage design, it can be observed that the sum rate maximization digital beamforming design approach the capacity of the broadcast equivalent channel, which is again equal to its uplink multiple access equivalent channel dual in information theory. Since the sum rate of dual multiple access channel is lower bounded by the sum rate of users with equal power allocation per RX per stream, using Jensen inequality, the sum rate is lower bounded by

$$\frac{1}{L}\sum_{\ell=1}^{L}\sum_{k=1}^{K}R_{\pi(\ell,k)}[\ell]$$
$$\geq\frac{1}{KL}\sum_{\ell=1}^{L}\sum_{k=1}^{K}\log\det\left(\frac{P}{Lr}\mathbf{H}_{\mathrm{eq},\pi(\ell,k)}[\ell]\mathbf{H}^{\mathrm{H}}_{\mathrm{eq},\pi(\ell,k)}[\ell]+\mathbf{I}\right)$$

According to (3) and $\mathbf{F}_{\mathrm{RF}} = [\mathbf{F}_{\mathrm{RF},1},\cdots,\mathbf{F}_{\mathrm{RF},K}]$, based on inequality $|\mathbf{A}+\mathbf{B}| \geq |\mathbf{A}|+|\mathbf{B}| \geq |\mathbf{A}|$, $\mathbf{A},\mathbf{B}$ are Hermitian positive semi-definite matrices, we can lower bound the average wideband sum rate as the object function in (P3).

Apply the same procedure of codebook constraint relaxation, SDR optimization, randomization and quantization as is described in Section III-B, we can obtain the RF design based on sum rate lower bound maximization. Due to space limit, we omit the algorithm chart. However, it is very similar to Algorithm 1 with only the optimization problem in step (6) and (9) being replaced by $(6')$ and $(9')$. Also the criterion in randomization procedure is to pick the candidate that maximize the sum rate lower bound.

$$\begin{aligned}\mathbf{X}_k^{[t]} &= \arg\max_{t_\ell,\mathbf{X}_k}\ \textstyle\prod_\ell^L t_\ell\\ \text{s.t.}\ &\mathrm{tr}(\mathbf{X}_k)\leq L_r\\ &\det\left(\tfrac{P}{L_r}\mathbf{X}_k\mathbf{H}^{\mathrm{H}}_{\pi(\ell,k)}[\ell]\mathbf{Y}^{[t]}_{\pi(\ell,k)}\mathbf{H}_{\pi(\ell,k)}[\ell]+\mathbf{I}\right)\geq t_\ell,\forall\ell\end{aligned}\tag{6'}$$

$$\begin{aligned}\mathbf{Y}_i^{[t+1]} &= \arg\max_{u_\ell,\mathbf{Y}_i}\ \textstyle\prod_\ell^L u_\ell\\ \text{s.t.}\ &\mathrm{tr}(\mathbf{Y}_i)\leq L_r\\ &\det\left(\tfrac{P}{L_r}\mathbf{H}_i[\ell]\mathbf{X}^{[t]}_{\gamma(\mathcal{U}_{\text{unique}}(i),\ell)}\mathbf{H}^{\mathrm{H}}_i[\ell]\mathbf{Y}_i+\mathbf{I}\right)\geq u_\ell,\forall\ell\end{aligned}\tag{9'}$$

One particular advantage for the proposed algorithms is that they lead to small signaling overhead for the RF beamforming design. In previous works [8], [13]–[17], the RF beamforming requires the TX to gather CSI for all scheduled users on all allocated subcarriers. However, in the proposed algorithms, the RF precoding sub-matrix $\mathbf{F}_{\mathrm{RF},k}$ is designed only with the CSI for the kth RX on all subcarriers, therefore the signaling is limited. In the extreme case when wideband scheduling strategy is used, i.e., the same RX is allocated over all the subcarriers, the RF precoding sub-matrix $\mathbf{F}_{\mathrm{RF},k}$ and RF combiners can be designed in a distributed manner at each scheduled user, based on their channel state information at the receiver (CSIR) on all subcarriers.

IV. Numerical Performance Analysis

In this section, we evaluate the performance of the MU-wideband algorithm proposed in Section III-B and III-C. The channel realizations are generated according to Section II-B. The operating central frequency is 28 GHz. Unless otherwise specified, the simulation settings are listed in table I.

L	K	N_t	N_r	L_t	L_r	N_s	σ^2
5	4	16	4	4	1	1	1

TABLE I: Simulation setting for a MU-wideband mmWave communication system.

The RF precoders and combiners are based on the 1D-subarray partition model [20]. Each column of the RF combiner is a length N_r DFT vector $\boldsymbol{v}$ with entries $\boldsymbol{v}_i = \frac{1}{N_r}e^{-\mathrm{j}\frac{2\pi}{\lambda}(i-1)d_V\cos\theta_{\text{etilt}}}, i=1,\ldots,N_r$. Each DFT beam vector has fixed beam direction θ_{etilt} which is selected from the set $\{\theta_{\text{etilt}}|\theta_{\text{etilt}}=\frac{\pi}{rN_r}(t-1/2), t=1,\ldots,rN_r\}$. The RF precoder codebook is defined respectively. In the simulations, we select antenna spacing $d_V=\frac{\lambda}{2}$, the oversampling ratio is $r=2$. Therefore, the RF precoder(combiner) codebook has $32(8)$ candidate beamforming vectors, respectively.

We consider two schedulers in the simulation. The first scheduler allows frequency multiplexing for different users on different subcarriers. The second scheduler is a wideband scheduler which allocate the same user on all subcarriers. Both schedulers adopt a simple uniformly random user scheduling.

Separate two stage hybrid precoding algorithms are simulated in this section. In order to have a fair comparison, all algorithms apply a sum rate maximization digital precoder/combiner design over each subcarrier in the baseband design.

Fig. 1 shows the average wideband sum rate as a function of the SNR. Two proposed algorithms are compared with a naive modified MU-narrowband algorithm in [9], [17], where the RF precoder and combiner are based on a single center frequency and is applied for all subcarriers. It can be observed that the proposed algorithm outperforms the naive modified modified MU-narrowband algorithm in [9], [17] in both wideband scheduling and frequency multiplexing case. When frequency multiplexing is applied, this gain is negligible since the user channels for the same RX over multiple subcarriers are highly correlated. Therefore, design the RF precoder and combiner base on a single frequency and apply them for all subcarriers will not impose very big mismatch. However, when frequency multiplexing is used, at 10dB SNR, we can find a $10\%(6\%)$ average wideband sum rate performance increasing between proposed sum rate lower bound maximization (max min equivalent channel gain) algorithm and modified MU-narrowband algorithm in [9], [17], respectively. It also reveals that wideband scheduling is more advantageous than frequency multiplexing. This confirms the conclusion in [8], stating that when the beam squint effect is not severe, it's more spectrum efficient to allocate the same user onto the entire band since the channels of different subcarriers are highly correlated.

Consider another convention simulation setting: wideband MU-MIMO mmWave system with all RXs having single antennas ($N_r = 1$) and wideband scheduling is used, the proposed algorithms are compared with the state of art algorithms in [8], [17]. Fig. 2 shows the average wideband sum rate as a function of the SNR in this conventional simulation

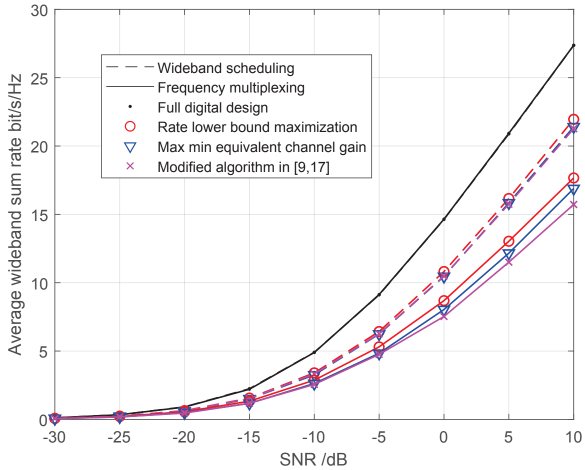

Fig. 1: Average wideband sum rate vs. SNR for different precoding algorithms for wideband multi-user MIMO mmWave communication system: multiple antennas RX, different scheduling strategies considered.

setting. We can conclude that all four algorithms have very small performance degradation compared with the full digital design. Our proposed algorithm has an about 2.7% averge sum rate performance degradation than algorithms in [8], [17]. However, algorithms in [8], [17] require CSI for all users on all subcarriers gathered at TX for the RF precoding design, which leads to very heavy signaling overhead. On the contrary, our proposed algorithms require much less signaling. Since wideband scheduling is assumed in this setting, the RF precoder can be designed in a distributed manner at each scheduled user, solely based on local CSI at the receiver on all scheduled subcarriers. Therefore, the proposed algorithms have good performance complexity trade-off.

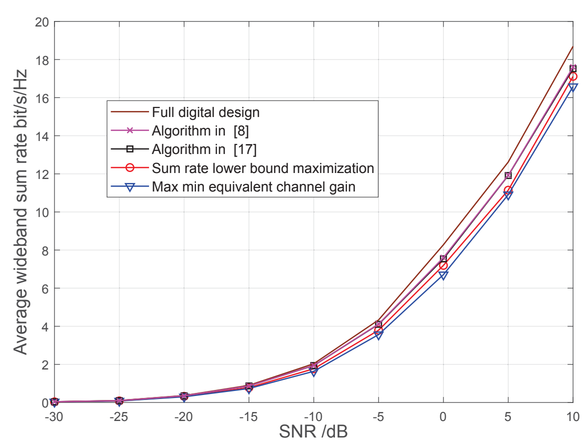

Fig. 2: Average wideband sum rate vs. SNR for different precoding algorithms for wideband multi-user MIMO mmWave communication system: wideband scheduling, single antennas RX considered.

V. Conclusion

In this paper we propose two RF beamforming designs for the downlink transmission of a wideband multi-user massive MIMO mmWave system. The algorithms are valid for different scheduling strategies and RX antennas/RF chains/data streams configuration. They have low complexity and require low CSI signaling overhead. In mutiple RX antennas setting, simulation shows that the proposed algorithms outperform the state of the art hybrid beamforming algorithms. In single RX antenna setting, simulations shows that the proposed algorithms approaches the state of the art design while the CSI signaling overhead is largely reduced.

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Algorithm 1 RF design for max-min equivalent channel gain

Input: scheduled users on each subcarrier $\pi(\ell,k), \forall \ell = 1,\cdots,L,\ \forall k = 1,\cdots,K$

Let $\mathcal{U} = \{\pi(\ell,k), \forall \ell, \forall k\}$, $\mathcal{U}_{\text{unique}}$ is the set $\mathcal{U}$ with all repeating elements removed. $|\mathcal{U}_{\text{unique}}|$ denotes the cardinality of set. Define the mapping function $\gamma(\mathcal{U}_{\text{unique}}(i),\ell) : \mathcal{U}_{\text{unique}} \times \{1,\cdots,L\} \mapsto \{1,\cdots,K\}$ which indicates RX $\mathcal{U}_{\text{unique}}(i)$ is the $\gamma(\mathcal{U}_{\text{unique}}(i),\ell)$th user on subcarrier ℓ, $\forall i = 1,\cdots,|\mathcal{U}_{\text{unique}}|$

Initialize RF combiner $\mathbf{W}^{[0]}_{\mathrm{RF},\pi(\ell,k)}$ for all scheduled users. Let $\mathbf{Y}^{[0]}_{\pi(\ell,k)} = \mathbf{W}^{[0]}_{\mathrm{RF},\pi(\ell,k)} \left(\mathbf{W}^{[0]}_{\mathrm{RF},\pi(\ell,k)}\right)^{\mathrm{H}}, \forall \ell, k$. Initialize $t = 0$

while Not converge **do**

  for $k = 1 : K$ **do**

$$\begin{aligned} \mathbf{X}^{[t]}_k = \arg\max_{f,\mathbf{X}_k}\ & f \\ \text{s.t.}\ & \operatorname{tr}(\mathbf{X}_k) \le L_r \\ & \operatorname{tr}\left(\mathbf{X}_k \mathbf{H}^{\mathrm{H}}_{\pi(\ell,k)}[\ell] \mathbf{Y}^{[t]}_{\pi(\ell,k)} \mathbf{H}_{\pi(\ell,k)}[\ell]\right) \ge f, \forall \ell \end{aligned}$$

  end for

  for $i = 1 : |\mathcal{U}_{\text{unique}}|$ **do**

$$\begin{aligned} \mathbf{Y}^{[t+1]}_i = \arg\max_{g,\mathbf{Y}_i}\ & g \\ \text{s.t.}\ & \operatorname{tr}(\mathbf{Y}_i) \le L_r \\ & \operatorname{tr}\left(\mathbf{H}_i[\ell] \mathbf{X}^{[t]}_{\gamma(\mathcal{U}_{\text{unique}}(i),\ell)} \mathbf{H}^{\mathrm{H}}_i[\ell] \mathbf{Y}_i\right) \ge g, \forall \ell \end{aligned}$$

  end for

  $t = t + 1$

end while

if rank constraints satisfied **then**

  $\mathbf{F}_{\mathrm{RF},k} = (\mathbf{X}^\star_k)^{\frac{1}{2}}, \mathbf{W}_{\mathrm{RF},\pi(\ell,k)} = \left(\mathbf{Y}^\star_{\pi(\ell,k)}\right)^{\frac{1}{2}}$

else

  Randomization procedure: Generate random Gaussian matrix $\mathbf{V}_1 \in \mathcal{C}^{N_t \times Lr}$, $\mathbf{V}_2 \in \mathcal{C}^{N_r \times Lr}$, each entries of $\mathbf{V}_1, \mathbf{V}_2$ are iid with distribution $\mathcal{N}_{\mathbb{C}}(0,1)$.

$$\begin{aligned} \mathbf{X}^\star_k &= \mathbf{U}_{\mathbf{X}^\star_k} \mathbf{\Lambda}_{\mathbf{X}^\star_k} \mathbf{U}^{\mathrm{H}}_{\mathbf{X}^\star_k} \\ \mathbf{Y}^\star_{\pi(\ell,k)} &= \mathbf{U}_{\mathbf{Y}^\star_{\pi(\ell,k)}} \mathbf{\Lambda}_{\mathbf{Y}^\star_{\pi(\ell,k)}} \mathbf{U}^{\mathrm{H}}_{\mathbf{Y}^\star_{\pi(\ell,k)}} \\ \mathbf{F}_{\mathrm{RF},k} &= \mathbf{U}_{\mathbf{X}^\star_k} \mathbf{\Lambda}^{\frac{1}{2}}_{\mathbf{X}^\star_k} \mathbf{V}_1 \\ \mathbf{W}_{\mathrm{RF},\pi(\ell,k)} &= \mathbf{U}_{\mathbf{Y}^\star_{\pi(\ell,k)}} \mathbf{\Lambda}^{\frac{1}{2}}_{\mathbf{Y}^\star_{\pi(\ell,k)}} \mathbf{V}_2 \end{aligned}$$

  Normalize each column of $\mathbf{F}_{\mathrm{RF},k}, \mathbf{W}_{\mathrm{RF},\pi(\ell,k)}$ such that they are unit vectors. Repeat N_{Rand} times for the $\mathbf{F}_{\mathrm{RF},k}, \mathbf{W}_{\mathrm{RF},\pi(\ell,k)}$ approximation and pick the one that yields the max-min equivalent channel gain.

end if

Find the codewords in the codebook which best approximate $\mathbf{F}_{\mathrm{RF},k}, \mathbf{W}_{\mathrm{RF},\pi(\ell,k)}$